The fundamental relation between ROA and ROE

K_e	Cost of Equity	E	Equity
K_d	Cost of Debt	D	Long term Debt
K_a	Cost of Asset	V	Value of Asset (V = D + E)

$$K_e.\frac{E}{V} + K_d.\frac{D}{V} = K_a$$

$$K_e.\frac{E}{V} = K_a - K_d.\frac{D}{V}$$

$$K_e = \frac{K_a}{\frac{E}{V}} - \frac{K_d.\frac{D}{V}}{\frac{E}{V}}$$

$$K_e = K_a.\frac{V}{E} - K_d.\frac{D}{V}.\frac{V}{E}$$

$$K_e = K_a.\frac{V}{E} - K_d.\frac{D}{E}$$

$$K_e = K_a.\left(\frac{D}{E} + \frac{E}{E}\right) - K_d.\frac{D}{E}$$

$$K_e = K_a.\left[1 + \frac{D}{E}\right] - K_d.\frac{D}{E}$$

$$K_e = K_a + K_a.\frac{D}{E} - K_d.\frac{D}{E}$$

$$K_e = K_a + \frac{D}{E}.[K_a - K_d]$$