

Properties of high frequency DAX returns: Intraday patterns, Jumps and their impact on subsequent volatility

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Abstract

This paper analyzes the behavior of the German DAX index intraday returns. We devote particular attention to three related empirical issues. First we provide an up-to-date characterization of the DAX intraday volatility patterns. They are mostly W-shaped with peaks at the opening, at 2.30pm and before the closing. We find some evidence suggesting that the implied volatility also follows some deterministic patterns over the trading day. Second we identify jumps in DAX returns. On jump days, they account on average for 15% to 25% of the daily variance. Jumps also tend to cluster and are not evenly distributed throughout the trading day. Third we estimate the impact of a price jump on volatility. We consider different proxies for volatility: absolute returns, implied volatility and realized volatility. Our results indicate that negative jumps trigger a strong upward correction in volatility. This correction starts just after a jump occurred and persists during up to 25 minutes. On the other hand, positive jumps seem to have a much less significant impact on volatility. These results hold for all volatility proxies but they are more significant when we consider the implied volatility.

JEL classification: G10; G12; G13

Keywords: Intraday patterns; price jumps; implied volatility; realized variance; asymmetric volatility.

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1 Introduction

In this paper, we describe and investigate some phenomena and regularities that emerge when observing high-frequency DAX returns. We do not focus on the *sign* of returns but rather on their *amplitude* (or *volatility*). The sign of returns should indeed be unpredictable according to the Efficient Market Hypothesis (Fama (1970)).¹ Furthermore, empirical evidence suggests that their amplitude displays some regularities (Cont (2001) and Granger & Poon (2002)).

The amplitude of intraday returns mainly depends on the interaction between two driving variables: the trading activity and the volume of information that reaches the market at a particular moment of the day. The first mostly determines the speed at which prices are updated, while the second has a direct effect on the amplitude of individual price changes. There is obviously a link between these variables. In the absence of significant news, less traders are active on the market. Similarly, the way new information is reflected in the prices depends on the number of active traders. There is a vast literature that directly models either the time duration between trades² or the process by which equilibrium prices are reached^{3,4}. Unlike these, we adopt what can be considered as a *reduced form* approach and directly study some particular features of high-frequency price changes. Our aim is to characterize the so-called volatility intraday patterns on the German market, to identify price discontinuities (jumps) and to analyze their statistical properties and their impact on volatility.

The way trading activity spreads throughout the day is our first object of study. Trading activity and the amplitude of price changes are linked together: in general, the higher the activity at a particular moment of the day, the higher is the intraday variance at that moment. The literature on intraday volatility patterns traces back to Wood *et al.* (1985) and was mostly concerned with US equity markets and FX markets as these are open 24 hours a day.⁵ These studies have shown that the patterns are typically U-shaped. The highest level of trading activity is reached just after the opening and before the market closes. During lunchtime, there are only few active traders and therefore the variance in this period remains rather low. The existence of these regularities have both theoretical and practical consequences. Notably, it is crucial to filter out intraday patterns from raw returns before trying to fit a parametric volatility model; otherwise the estimates might be severely biased (Andersen *et al.* (1999)). Moreover, intraday traders have concerns about very short-term moves and are obviously affected by higher intraday volatilities as they may lead to rapid losses.

¹ This is not exactly the case for high-frequency returns as they are known to exhibit a significant negative first-order autocorrelation. See, e.g., Andersen & Bollerslev (1997a) or Dacorogna *et al.* (1997).

² Two recent contributions to this literature are Fernandes & Grammig (2005) and Meitz & Teräsvirta (2006).

³ See, e.g., the survey article of Madhavan (2000).

⁴ A related literature, which traces back to Clark (1973), is concerned with the so-called mixture of distribution hypothesis. See also Tauchen & Pitts (1983) and Bauwens *et al.* (2006).

⁵ See Wood *et al.* (1985) and Harris (1986) for some early discussions on intraday patterns for equity markets; a recent contribution is Tian & Guo (2006), who provide an up-to-date literature review and also study intraday patterns on the Shanghai Stock Exchange. To the best of our knowledge, the only articles studying intraday patterns on the German market are Kirchner & Schlag (1998) and Ozenbas *et al.* (2002). For FX markets, see, e.g., Muller *et al.* (1990), Baillie & Bollerslev (1991), Dacorogna *et al.* (1993) and Andersen & Bollerslev (1997b).

New information impacts the market differently depending on its importance and whether it is expected or not. For instance, there are recurrent news that reach the market each day with a regular timing and deliver essentially the same kind of information. Notably, the overnight evolution of US and Asian markets has an impact on the opening of European markets. Similarly, the mid-afternoon period is characterized by the release of information from the USA and the opening of the NYSE and the Nasdaq. Because of its recurrent nature, this kind of information directly affects the intraday volatility patterns. On the other hand, there are many unexpected pieces of information about firms, economic situation or investor sentiments, that have a unique and important effect on prices. The release of such news often generates price jumps. Our second objective is to identify and characterize those jumps on the German market. We make use of recent advances in the study of so-called stochastic volatility semimartingale (SVSM) processes to identify the jumps.⁶ We then study their occurrences, size and timing. Jumps are typically located in the tails of the return distribution and are thus very relevant for risk management purposes. They also have a strong impact on the ability to properly hedge a derivative position as it is not possible anymore to replicate it perfectly.

Our third objective is to analyze the relationship among returns and changes in volatility when there are price discontinuities. Knowledge of this relationship is a key ingredient for effective option pricing and it might also help establishing innovative trading strategies. In the continuous case (i.e. in the absence of jumps), this relationship has already attracted much interest from researchers. Volatility changes have typically been found to be negatively and asymmetrically related to returns (Black (1976) and Christie (1982)). Two theories have been advanced for explaining this effect: the leverage and the feed-back explanations. The first assumes that past returns have a predictive power on futures volatility changes, while the other implies the opposite chain of reactions.⁷ The causal relationship remains mostly unclear when considering daily data (see Bouchaud *et al.* (2001)). When switching to a high-frequency framework, one observes that the relation is mostly return-driven. Previous returns have an inverse impact on subsequent volatility changes (Bollerslev *et al.* (2006) and Masset & Wallmeier (2007)). The relationship between price jumps and subsequent volatility changes has not yet been investigated. Though an often encountered (sometimes explicit but mostly implicit) assumption is that both price and volatility jump simultaneously. We test if this assumption holds. In particular, we analyze the impact of a price discontinuity on contemporaneous and subsequent volatility changes. If a jump is expected to have a persistent impact on the magnitude of subsequent returns and if the investors are fully rational, the implied volatility should jump (almost) simultaneously.

For our inferences, we make use of a high-quality database, which covers all transactions involving options and futures on the German market from January 1995 to December 2005.⁸ There is a one-to-one relationship between future prices and actual index levels; hence we can estimate DAX levels with a very high level of precision. This is obviously crucial for the analysis of intraday

⁶ See, e.g., Andersen *et al.* (2001b), Andersen *et al.* (2001a), Barndorff-Nielsen & Shephard (2002), Andersen *et al.* (2006b) and Bandi & Russell (2007).

⁷ For a detailed discussion of these two theories, see, e.g., Bekaert & Wu (2000), Bollerslev *et al.* (2006) and Masset & Wallmeier (2007).

⁸ Over this period, DAX futures and options have represented the highest trading volume among all stock index derivatives in Europe.

patterns and the identification of price discontinuities. We compute the implied volatility from option prices. Implied volatilities present several advantages over other volatility proxies (like squared returns or realized variance). They do not depend on actual index returns and they allow us to accurately determine the point in time when changes in volatility occur. It is thus possible to gauge precisely the impact of a price jump on the volatility process.

This paper contributes to the existing literature in several ways. First, we provide a precise and up-to-date description of the intraday patterns for squared returns and for implied volatility on the German market. The patterns for squared returns are mostly *W*-shaped: returns are especially large at the opening, at 2:30 p.m. and at the closing. We also find clear-cut patterns for the implied volatility; they have a *J*-shape: implied volatility is typically lower during the lunch than during the rest of the day and it usually reaches a high when the market closes. Second, we characterize the occurrences of jumps and their statistical properties. On jump days, price discontinuities account on average for 15% to 25% of the daily return variance and, on some days, a jump might even explain as much as 80% of the daily variance. As already noticed by Andersen *et al.* (2007), jumps tend to cluster. Furthermore, the probability of a jump seems to be higher in the early morning, at 2:30 p.m. and in the late afternoon. Third, we show that the impact of price discontinuities on both implied and realized volatility is considerable. Negative jumps trigger a strong increase in volatility. In particular, about 20 to 25 minutes are needed by the implied volatility process to fully incorporate the effects of such a jump. On the other hand, positive jumps have only a limited impact on volatility.

Our paper is organized as follows. In the next section, we describe our dataset and discuss some issues related to high-frequency frictions. The methodology used to estimate intraday patterns and to identify price discontinuities is presented in section 3. Intraday patterns for squared returns and for implied volatility are discussed in section 4. In section 5, we study the number of jumps, their importance and their timing. Section 6 is devoted to the analysis of the impact that a jump has on volatility. Section 7 concludes.

2 Data

Our data come from the joint German and Swiss options and futures exchange, Eurex.⁹ The Eurex is the world's largest futures and options exchange and is jointly operated by Deutsche Börse AG and SWX Swiss Exchange. The database contains all reported transactions of options and futures on the German stock index DAX from January 1995 to December 2005. The average daily trading volume of DAX options (ODAX) and futures (FDAX) in December 2005 was of 166,886 and 117,388 contracts, respectively. The options are European style. At any point in time during the sample period, at least eight option maturities were available. However, trading is heavily concentrated on the nearby maturities. The contract values amount to 5 euros (ODAX). Trading hours changed several times during our sample period, but both products were traded at least from 9:30 a.m. to 4:00 p.m.

⁹ We are very grateful to the Eurex for providing the data.

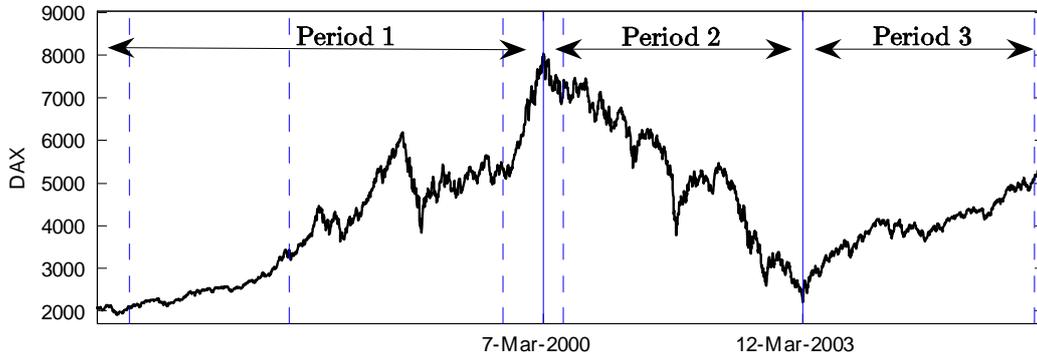


Figure 1: Period definition.

We split the observation period in eight subsamples according to two criteria: (i) market conditions (bull or bear) and (ii) trading hours. This has no impact on the identification of jumps because the test statistics used to detect jumps consider each day separately. It also makes no sense to use data that have different opening or closing hours when estimating intraday patterns. Furthermore, this gives us the opportunity to test if intraday patterns and jumps share the same features in different market conditions. Between 1995 and 2005, we identify three broad periods of either bullish or bearish market, periods 1 to 3 in figure 1. The first period corresponds to the long bull market between 1995 and 2000, which ended with the burst of the tech bubble on March 10th, 2000. The second period, which covers years 2000 to 2003, saw the market suffer severe losses, caused by the post-bubble correction and was further reinforced by the September 11th terrorist attacks. The beginning of the Iraqi war on March 20th, 2003 marked the beginning of a new bull market, which is still running (as of December 2005). We finally split once again each period such that all observation days in a subperiod have the same trading hours.¹⁰

2.1 Computation of DAX and implied volatility levels

We use the relationship between futures and spot prices to estimate the underlying DAX index levels.¹¹ Similarly, we calculate the implied volatility from the Black & Scholes (1973) formula. In order to obtain correct implied volatilities, it is crucial to accurately match the option price and the corresponding underlying price. As we use time-stamped tick-by-tick data, matching of option and DAX index levels is straightforward. We apply the method of Hafner & Wallmeier (2001) to account for dividend effects and to ensure put-call-parity consistent estimates of implied volatilities. We remove all options that violate the arbitrage bounds or have implied volatilities

¹⁰ The start and end dates of each subperiod are the followings: 01/01/1995 - 05/19/1995 (Period 1A), 05/22/1995 - 03/27/1997 (1B), 04/01/1997 - 09/17/1999 (1C), 09/20/1999 - 03/07/2000 (1D), 03/08/2000 - 06/01/2000 (2A), 06/02/2000 - 03/12/2003 (2B), 03/13/2003 - 11/18/2005 (3A), 11/21/2005 - 12/30/2005 (3B).

¹¹ We use futures to infer DAX levels as some evidence suggests that futures should react more rapidly to news than the underlying (Hasbrouck (2003)). Furthermore, there is a direct link between options and futures as the latter are usually used to hedge the firsts (Liu *et al.* (2007)).

higher than 150% (Hafner & Wallmeier (2001)). We consider only options with a time-to-maturity longer than 10 calendar days to avoid expiration-day effects and because estimates for implied volatility are known to be unstable for shorter maturities.¹² Options with a very distant maturity are much less liquid and it is difficult to account properly for term structure effects. Therefore we keep only options that have a time-to-maturity no longer than 3 months. There are several difficulties to accurately estimate the implied volatility of deep in-the-money (ITM) and deep out-of-the-money (OTM) options. Such options are prone to be affected by errors in measurement (Hentschel (2003)). Therefore we do not use options with a moneyness lower than 0.85 or larger than 1.15. This moneyness interval is relatively wide as we want to have the largest number of valid observations in our sample.

Implied volatilities are not constant across moneyness levels and also vary with the time-to-maturity. OTM put options typically have higher implied volatilities than at-the-money (ATM) put options. This phenomenon is known as the volatility smile or volatility smirk in the finance literature (see, e.g., Rebonato (2004)). The existence of an implied volatility term structure is a related issue: the implied volatility tends to decrease when the option approaches maturity. Consequently, one cannot directly use the implied volatilities estimated from options with different features (maturity and strikes) to construct a homogeneous time-series of implied volatility for the underlying. This implies that a change in implied volatility between two subsequent trades can either be due to a change of its fundamental level or to a different moneyness or time-to-maturity. Hereafter, we detail how we proceed to rule out smile and term structure effects.

SMILE CORRECTION

We first estimate the smile structure each day following the cubic regression approach described in Hafner & Wallmeier (2001) and Hafner & Wallmeier (2007) and, then, we use the fitted smile function to remove the impact of moneyness on implied volatilities. More specifically, let K denote the strike price of an option with time to maturity $T - t$. Each trade is assigned a moneyness according to:

$$M(t, T, K) = \frac{\ln\left(\frac{K}{F_t(T)}\right)}{\sqrt{T-t}},$$

where $F_t(T)$ is the forward price at time t for maturity T . Thus, ATM options are characterized by a moneyness of 0. Suppressing the arguments of moneyness, we chose the cubic regression function:

$$\sigma = \beta_0 + \beta_1 M + \beta_2 M^2 + \beta_3 D \cdot M^3 + \varepsilon, \quad (1)$$

where σ is the implied volatility, β_i , $i = 0, 1, 2, 3$ are regression coefficients, ε is a random error, and D is a dummy variable defined as:

$$D = \begin{cases} 1 & , \quad M > 0 \\ 0 & , \quad M \leq 0 \end{cases} .$$

¹² These points are discussed in De Jong & Donders (1997). In other studies, the last 7 to 10 days are usually removed. For instance, Dennis *et al.* (2006) removed options with a maturity of one week or less; Masset & Wallmeier (2007) took only options with at least 10 days to maturity.

The dummy variable accounts for an asymmetry of the pattern of implied volatilities around the ATM strike ($M = 0$).

Let $\sigma_{imp}(M, t)$ denote the implied volatility of an option with moneyness M traded at time t . Then, the corresponding ATM implied volatility $\sigma_{imp}^{ATM}(t)$ is calculated as

$$\sigma_{imp}^{ATM}(t) = \sigma_{imp}(M, t) - \left[\widehat{\beta}_1 M + \widehat{\beta}_2 M^2 + \widehat{\beta}_3 D \cdot M^3 \right], \quad (2)$$

where $\widehat{\beta}_i$ are the estimated regression coefficients.

TERM STRUCTURE CORRECTION

We estimate the term structure of implied volatility using the linear regression model:¹³

$$\sigma_{imp}^{ATM}(t) = \alpha_0 + \alpha_1(T - t) + \alpha_2 D_2 \cdot (T - t) + u, \quad (3)$$

where α_i , $i = 0, 1, 2$ are regression coefficients, u is a random error, and D_2 is a dummy variable defined as:

$$D_2 = \begin{cases} 1 & , \quad T - t > 60 \\ 0 & , \quad \text{otherwise} \end{cases} .$$

The dummy variable gives more flexibility to the specification as it allows the trend of the term-structure to be different for options with the longest time-to-maturity. The implied volatility corresponding to a 30-day constant time-to-maturity option can then be calculated as:

$$\sigma_{imp}^{ATM,CTtM}(t) = \sigma_{imp}^{ATM}(t) - [\widehat{\alpha}_1(T - t - 30) + \widehat{\alpha}_2 D_2 \cdot (T - t - 30)], \quad (4)$$

where $\widehat{\alpha}_i$ are the estimated regression coefficients.

2.2 High-frequency frictions

Previous operations leave us with two time-series, one of implied volatility and another of DAX levels. Both series are recorded at irregular time intervals. In order to get homogeneous time-series, we resample the data by aggregating them over a particular sampling frequency. There is always a trade-off between the amount of information that can be inferred from the data and the noise that they might contain (see Hansen & Lunde (2006)). In order to keep as much information as possible from the original data, the highest possible sampling frequency should be considered. However, this is at a cost as it exacerbates the impact of high-frequency frictions and thus augments the noise-to-signal ratio.¹⁴ DAX futures are very liquid and have a small

¹³ We also consider non linear models but the final results remain mostly unaltered.

¹⁴ For a discussion of optimal sampling procedures, see, e.g., Ait-Sahalia *et al.* (2005), Zhang *et al.* (2005) and Bandi & Russell (2008).

bid-ask spread. Thus, a very high sampling frequency could be selected. Unfortunately, DAX options are less liquid and have a larger bid-ask spread. We therefore decide to sample the data each 5 minutes. This consensual choice is indeed quite common in the literature (see, e.g., Bollerslev *et al.* (2006)).

The estimated DAX level for each 5-minute interval is set equal to its average over the last minute of the interval. In very few cases, there is no observation in the last minute of an interval and thus the DAX level cannot be estimated, implying that the corresponding return cannot be computed. This is an issue as the procedure we employ to identify price discontinuities (see section 3.2) requires a complete series of returns. In order to solve this, we complete the original series and estimate the prevalent DAX level by interpolating between the average DAX levels over the previous and the next minute with valid observations.¹⁵ As stated above, options are less liquid than futures: in many intervals, there is no observation at all and consequently it is not possible to calculate an implied volatility. We do not replace these missing values for two reasons. First, we do not necessarily need a complete series of implied volatilities as they are not required for identifying price jumps. Second, when analyzing the relationship among DAX jumps and volatility changes, we want to avoid spurious results caused by the presence of fictitious (interpolated) values. Consequently, to conduct our inferences in the next sections, we employ only the intervals in which we have a valid observation for the implied volatility.

Log-returns of implied volatility and of the underlying stock index are calculated as:

$$R_{v,t_j} = \ln [\sigma_{imp}^{ATM}(t_j)] - \ln [\sigma_{imp}^{ATM}(t_{j-1})] \quad \text{and} \quad R_{S,t_j} = \ln [S(t_j)] - \ln [S(t_{j-1})],$$

where $\sigma_{imp}^{ATM}(t_j)$ is the implied volatility and $S(t_j)$ denotes the index level in the j -th 5-minute interval on day t . R_{v,t_j} is only calculated if we have an observation at times t_j and t_{j-1} . We remove overnight returns (Gwilym & Buckle (2001)) and the first return of each day (Bollerslev *et al.* (2004)).

We eventually devote some consideration to the impact of the bid-ask spread on returns. As is well known, the bid-ask bounce leads to a negative first order autocorrelation of returns (Roll (1984)). This spurious autocorrelation comes from successive trades, where one is executed at the bid, the other at the ask price. As already stated, futures have a narrower bid-ask spread than options. It is thus not surprising to find out that this effect is very pronounced for volatility returns but absent from DAX returns.¹⁶ We use the standard method to remove spurious autocorrelation from volatility returns only. This method consists of filtering returns with a MA(1) process (see, e.g., Stephan & Whaley (1990), Easley *et al.* (1998) and Gwilym & Buckle (2001)):

$$R_{v,t_j} = \mu_v + e_{v,t_j} - \theta_v e_{v,t_{j-1}},$$

where R_{v,t_j} is the observed implied volatility return, μ_v the unconditional mean of the observed returns series, θ_v the moving average coefficient, and e_{v,t_j} the innovation of the process. Since the innovations from the MA(1) process are uncorrelated, we can use them as bid-ask bounce corrected returns. Through this correction, raw implied volatility returns R_{v,t_j} are transformed into adjusted log-returns r_{v,t_j} , i.e. $r_{v,t_j} = e_{v,t_j}$.

¹⁵ A robustness check shows that this procedure has no impact on the estimation of the intraday patterns.

¹⁶ First order autocorrelation coefficients are -0.4159 and 0.0170 for volatility returns and DAX returns.

3 Estimation of the intraday patterns and jump identification

3.1 Methodology for estimating intraday patterns

Intraday patterns are just a graphical and quantified representation of how the activity on the market fluctuates during the trading day. The level of activity depends on the number of active traders and the quantity and type of information reaching the market at a particular time. When there are more active traders, prices change more quickly and the returns' variance computed over a fixed-length interval increases.¹⁷ This implies that intraday patterns can be estimated on the basis of the amplitude of price changes computed at different moments of the day. These patterns can also be considered as a time deformation in the sense of Clark (1973) as they deliver some information about the speed at which business time flows. When dealing with a return series sampled in the usual clock time, it is crucial to filter intraday patterns out from raw returns in order to avoid biases in the modelling of conditional volatility (Andersen *et al.* (1999)).

To study and estimate intraday patterns, their influence has to be isolated from the original series of returns. A usual assumption is that observed high-frequency returns can be split into two parts: an unpredictable (innovation) component and a volatility component. The volatility component itself is the product of both the prevalent daily volatility and an intraday volatility factor. The return in interval j on day t can thus be expressed as:

$$R_{t_j} = \sigma_t \cdot \lambda_j \cdot z_{t_j}, \quad (5)$$

where σ_t is a measure of the latent volatility on day t , λ_j is a factor that accounts for the proportion of a trading's day return variance that is attributed to interval j , and z_{t_j} is an innovation term with mean zero and variance one. Intraday patterns should reflect some typical, inherent, characteristic of the activity flow during trading hours; therefore they are not expected to change from one day to another. We define standardized returns r_{t_j} as $r_{t_j} = \frac{R_{t_j}}{\sigma_t}$, where σ_t can be estimated either from a parametric model (e.g. a Stochastic Volatility or a GARCH model) or a non-parametric one (e.g., the realized volatility, see section 3.2). Assuming no covariation among σ_t , λ_j and z_{t_j} , the variance of R_{t_j} can be calculated as $Var(R_{t_j}) = \sigma_t^2 \cdot \lambda_j^2$ (as $Var(z_{t_j}) = 1$) and the variance of r_{t_j} is:

$$Var(r_{t_j}) = \lambda_j^2. \quad (6)$$

Further, z_{t_j} has mean zero and thus the variance can be equally computed from:

$$Var(r_{t_j}) = E(r_{t_j}^2) = N^{-1} \sum_{t=1}^N r_{t_j}^2 = \lambda_j^2. \quad (7)$$

¹⁷ Unless returns from successive trades cancel out, the overall price variation will indeed be higher. Note also that this does not mean that the volatility of a single trade should increase.

This expression suggests a simple way for estimating the intraday volatility factors. As an alternative, one may instead focus on the expression $|r_{t_j}|$, which is less influenced by extreme values. If z_{t_j} comes from a normal distribution, we have that $E(|r_{t_j}|) = \sqrt{\frac{2}{\pi}}\lambda_j$ and thus we get $\hat{\lambda}_j = \sqrt{\frac{\pi}{2}} \cdot E(|r_{t_j}|)$ as an estimate for λ_j . Nevertheless, the assumption that the innovations z_{t_j} are normally distributed is very disputable. Therefore, we opt for an estimation of the intraday patterns based on (7).

The summation in (7) considers a set of N successive days; the larger N , the more precise the estimate for $\hat{\lambda}_j$ will be. Though, if the patterns are not stable through time¹⁸, the complete sample has to be split into homogeneous subsamples. Eventually, the factors' estimates remain noisy and sensitive to outliers. Some erratic changes in the patterns when moving from an interval j to the next interval $j + 1$ do not have much economic content. They can be caused by some large returns in a few days, either in interval j or $j + 1$. Therefore, it is preferable to smooth the original estimates. Andersen *et al.* (2001c) recommend Fourier flexible functions (FFFs):¹⁹

$$\lambda_j = \sum_{k=0}^K \mu_k j^k + \sum_{i=1}^D \alpha_i I_{j=d_i} + \sum_{p=1}^P [\delta_p \cos(\frac{2\pi j p}{N}) + \delta_p \sin(\frac{2\pi j p}{N})]. \quad (8)$$

The FFF model (8) consists of three parts. The first is based on a polynomial structure, which aims at modelling the general trend of the patterns over the trading day. If assuming a unique homogeneous trend for the whole trading day seems too unrealistic, the polynomial can be broken in several subpolynomials in order to account for different trading regimes. The second part shows a summation of dummies; their purpose is to model particular phases of the trading day which exhibit unusual characteristics and are thus difficult to capture through the polynomial part (e.g. the opening or the closing of the market). The last part is based on a series of sinusoidal functions, which work best when the series to be modelled exhibits cyclical behaviors. To obtain a smooth estimate of the intraday patterns, we further add an error term on the right hand-side of model (8) and then estimate the complete model by Ordinary Least Squares (OLS). Robust standard errors for the coefficients can be obtained using the Newey & West (1987) method.²⁰

¹⁸ This would be the case either if the trading hours in the home country or abroad (e.g. in the USA) or if the investors' trading habits have changed over time.

¹⁹ See also Andersen & Bollerslev (1997b).

²⁰ Dacorogna *et al.* (1993) propose another way to deal with intraday patterns. They suggest resampling the data in so-called business time. In their setting, time-intervals are defined as some proportion of the daily returns' quadratic variation. This procedure mechanically removes the effects of intraday patterns.

3.2 Realized variance and identification of jumps

Volatility is said to be latent as it is not directly observable. Nevertheless, recent advances in the theory of quadratic variation have demonstrated that it can be estimated in a robust manner.²¹ The realized variance on day t is defined as:

$$RV_t = \sum_{j=1}^M R_{t_j}^2, \quad (9)$$

where R_{t_j} is the j -th intraday return on day t . In a given day, the number of intervals sums to M . If this number tends to infinity ($M \rightarrow \infty$), i.e. if the sampling frequency is increased such that the interval between two return observations become infinitesimal, the realized variance becomes asymptotically equivalent to the quadratic variation of the process:

$$RV_t \xrightarrow{M \rightarrow \infty} QV_t, \quad (10)$$

where QV_t is the quadratic variation of the process. Furthermore, if the underlying does not exhibit any discontinuity, QV_t will be an unbiased estimator of the integrated variance, IV_t :

$$IV_t = \int_{t-1}^t \sigma^2(s) ds. \quad (11)$$

However, with jumps in the process, QV_t will equal the integrated variance *plus* the sum of squared jumps, i.e.:

$$QV_t = \int_{t-1}^t \sigma^2(s) ds + \sum_{n=1}^{N_t} \kappa_{t_n}^2, \quad (12)$$

where N_t is the number of jumps on a given day ($N_t \leq M$) and κ_{t_n} is the size of the n -th jump. In order to isolate the jump component from the quadratic variation, we need an estimator of the integrated variance, which remains consistent even in the presence of jumps in the process. Barndorff-Nielsen & Shephard (2004) propose another measure, namely the bipower variation (as opposed to the simple power or quadratic variation):

$$BV_t = \mu_1^{-2} \sum_{j=2}^M |R_{t_{j-1}}| |R_{t_j}|, \quad (13)$$

where $\mu_1 = \sqrt{\frac{2}{\pi}} = E(|Z|)$ denotes the mean of the absolute value of standard normally distributed random variable Z .

²¹ See, e.g., Andersen *et al.* (2001b), Andersen *et al.* (2001a), Barndorff-Nielsen & Shephard (2002), Andersen *et al.* (2006b) and Bandi & Russell (2007).

More generally,

$$\mu_a = \frac{2^{a/2}\Gamma(1/2(a+1))}{\Gamma(1/2)} \equiv E(|Z|^a), \quad (14)$$

where Γ is the Gamma function. Barndorff-Nielsen & Shephard (2004) further prove that:

$$BV_t \xrightarrow[M \rightarrow \infty]{} IV_t. \quad (15)$$

The intuition behind this result is the following (Huang (2004)). A jump might occur either in t_{j-1} or in t_j but not in t_{j-1} and in t_j . This is because the number of jumps in a given day is finite. Hence the probability to observe two successive jumps is zero when the number of partitions goes to infinity (i.e. when $M \rightarrow \infty$). Jumps may still enter into the calculation of the bipower variation through the cross-product $|R_{t_{j-1}}| |R_{t_j}|$; nevertheless, their contribution to the BV_t process also becomes negligible when M goes to infinity. This is the case for two reasons: (i) there is only a finite number of jumps a day and (ii) the magnitude of the returns from the continuous part of the process tends to zero when M goes to infinity. Consequently, as the number of partitions increases ($M \rightarrow \infty$), the contribution of the jump component to BV_t vanishes. Moreover, when the sampling frequency is very high (i.e. when $M \rightarrow \infty$), returns from successive intervals become increasingly close to each other and, therefore, BV_t consistently estimates the integrated variance.

The result (15) implies that one can estimate either the part of the variance due to the continuous volatility component or the part due to the jump component by considering BV_t or by taking the difference between RV_t and BV_t .²² Obviously, this difference, computed from empirical data, can be negative. This is counter-intuitive as the variance has to be positive. Thus we set the daily variance due to jumps JV_t equal to $\max(0, RV_t - BV_t)$.²³

Using the procedure described above, one invariably finds that there are frequent but mostly small or even negligible jumps in the return process. To test if a *significant* jump occurred during a given day we use the test statistics derived by Barndorff-Nielsen & Shephard (2004), Barndorff-Nielsen & Shephard (2006) and Andersen *et al.* (2007). We adopt the logarithmic test specification of Huang & Tauchen (2005) and Andersen *et al.* (2006a):

$$W_t = \frac{\ln RV_t - \ln BV_t}{\sqrt{\frac{1}{M}(\mu_1^{-4} + 2\mu_1^{-2} - 5) \max(1, \frac{TP_t}{BV_t^2})}}, \quad (16)$$

and TP_t is the realized quarticity, which can be estimated consistently using either a tri- (Andersen *et al.* (2007)) or a quadripower quarticity (Barndorff-Nielsen & Shephard (2004)). We focus on the tripower quarticity, which is estimated through the following formula:

²² It is possible to estimate the variance of the Brownian increments using multipower variations:

$$PV_t = \mu_p^{-2} \sum_{j=2/p}^M \prod_{k=0}^{2/p} |R_{t_{j-k}}|^p.$$

As long as the returns are raised to a power inferior to two ($p < 2$), PV_t will be a consistent estimator of the variance of the continuous part of the process (Barndorff-Nielsen & Shephard (2004)).

²³ Barndorff-Nielsen & Shephard (2004) and Andersen *et al.* (2007) adopt the same adjustment.

$$TP_t = M\mu_{4/3}^{-3} \sum_{j=3}^M |R_{t_{j-2}}|^{4/3} |R_{t_{j-1}}|^{4/3} |R_{t_j}|^{4/3}. \quad (17)$$

Under the null of absence of jumps, the test statistics W_t from (16) is asymptotically standard normally distributed.

4 Intraday volatility patterns

4.1 Intraday patterns for squared returns

The intraday volatility factors $\widehat{\lambda}_j$ have been estimated on the basis of equations (5) and (6). They are reported in figure 2. Full lines show the fit from the FFF regression model (8), which has been specified as follows. First, the original polynom has been broken into three subpolynoms to account for different trading regimes, which correspond to (1) the morning; (2) the lunch time and the first part of the afternoon (i.e. before US-opening); and (3) the rest of the day. Second, two dummies have also been added to the regression model in order to account for the first and the last interval of the trading day. The order of the polynomials has been set to five. This ensures a satisfactory trade-off between the smoothness of the fitted intraday patterns and the risk of overfitting.²⁴ As in Andersen & Bollerslev (1997b), we leave out the sinusoidal part in (8).

Many news are released before the market opens. As a consequence, the first minutes are highly nervous. The activity remains intense during the next few intervals and then begins to slightly decrease. It typically reaches a low between 12:00 a.m. and 1:00 p.m. In Europe, the mid-afternoon is usually rather nervous because of information releases from the USA. In particular, news about the coming US-opening leads the intraday volatility to skyrocket between 2:30 p.m. and approximately 2:45 p.m. After that, the market calms down for a short while. When the closing time is approaching, investors trade again very intensively. This eventually brings the intraday volatility to new highs.

The patterns we get for the different periods look very similar. The precision of the fitted patterns obviously depends on the number of observation days in each period. It seems that intraday volatility patterns are neither affected by investor sentiments nor by market conditions. For instance, period 2-B was strongly bearish (the index lost about 70% of its value), while period 3-A was extremely bullish (the index more than doubled during this period). Though the patterns for these two periods remain very similar.

²⁴ For instance, Andersen *et al.* (2001c) use a third order polynomial. Nevertheless, they use FX data, which display much simpler and smoother U-shaped patterns. As a matter of comparison, we also run the regression models with third order polynomials. Overall very similar shapes emerge but the ability of the model to fit peaks in activity is severely weakened.

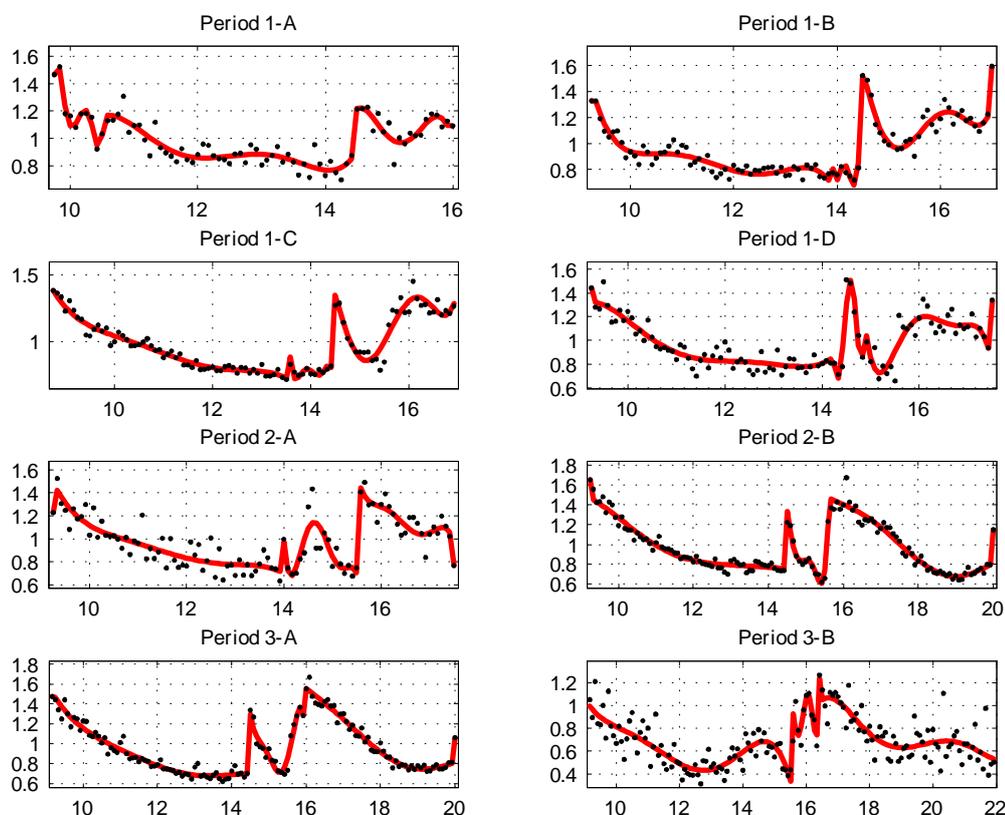


Figure 2: Intraday patterns for squared returns. The different panels show the patterns for each period of the sample.

4.2 Intraday patterns for implied volatility

As a matter of comparison, we check if the implied volatility might also display some kind of intraday patterns. We consider each period separately and proceed as follows. For each day, we estimate a midday implied volatility level, which is defined as the average implied volatility observed between 12:00 a.m. and 1:00 p.m. This midday estimate is then used to standardize the implied volatility levels in the other 5-minute intervals of the day. This standardization allows to directly compare the implied volatility measured in the same interval in different days. It thus permits to calculate for each period an average standardized implied volatility for each 5-minute interval.

Figure 3 displays the intraday patterns for implied volatility. The full line shows the fit from a fifth order polynomial regression (without any dummy or break). The implied volatility tends to decrease just after the opening. It then stabilizes during the second part of the morning and finally raises again during the whole afternoon. These patterns are clear-cut but remain less pronounced than the ones we get for the squared returns: the highest point of the intraday patterns is about 1% to 1.5% larger than the lowest one. Nevertheless this observation might

have important implication for option pricing as it means that, everything else held constant, the same option could well be cheaper when bought between 10:00 a.m. and 11:00 a.m. than in the late afternoon. In particular if the option is OTM, the price difference might be substantial. Consider for instance a call option written on an index that currently quotes at 8,500 points. The interest rate amounts to 3.5% yearly. The option strikes at 9,000 points and has a time-to-maturity of 30 calendar days. The implied volatility level is of about 20% at 10:00 a.m. According to the Black and Scholes formula [1973], the option price may increase by as much as 14% after a 1% increase in implied volatility (i.e., from 20% to 20.2%).

In order to quantify the importance of this effect from an economic viewpoint, we build a trading strategy based on a strangle. This is an indirect way to invest in volatility. We purchase simultaneously both an OTM call and an OTM put at 10:00 a.m. The position is closed at 4:00 p.m. This strategy has been implemented over the last five years of the sample. By the end of 2005, we achieve a raw return of about 0.62% per day. This number is much larger than the average daily return of the DAX over the same period but it remains lower than the effective transaction costs on the EUREX; in particular, the bid-ask spread is typically larger than 1% for OTM options.

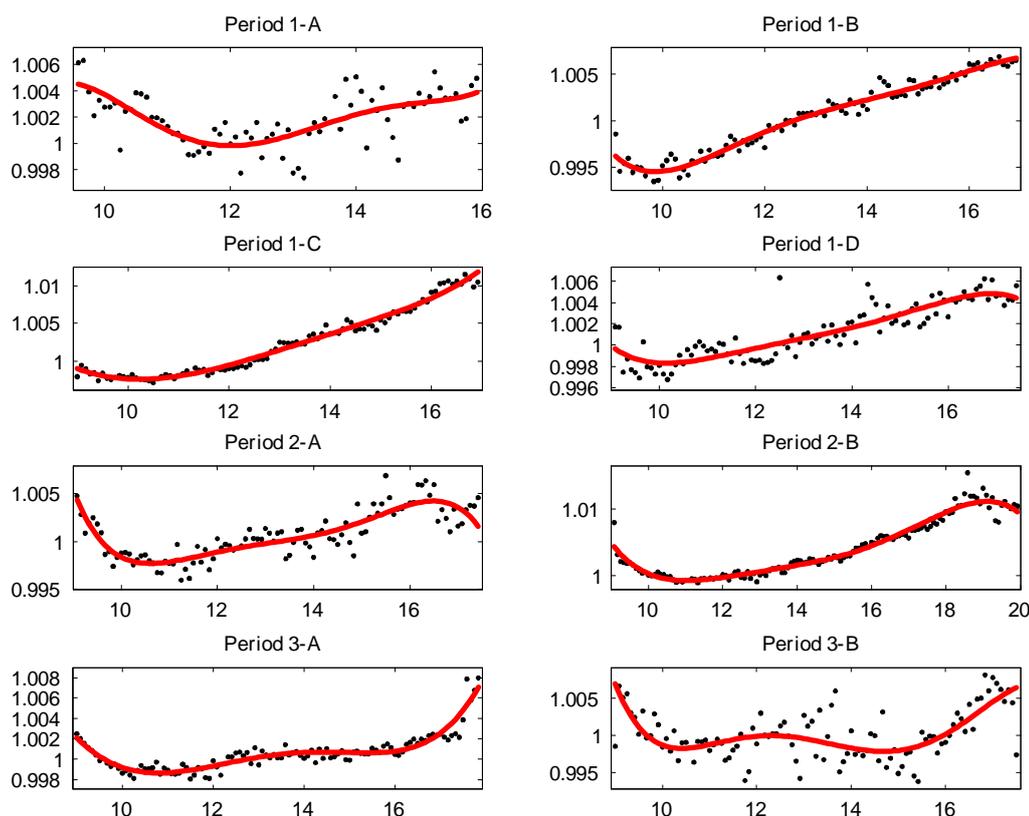


Figure 3: Intraday patterns for implied volatility. The different panels show the patterns for each period of the sample.

5 Price discontinuities

5.1 Number and importance of jumps

Table 2 reports descriptive statistics of jump occurrences. In order not to confuse a genuine jump with a burst of activity, intraday patterns have been preliminary filtered out from raw returns. We follow Huang & Tauchen (2005) and consider primarily the 99% and 99.9% confidence levels to identify price discontinuities. As a matter of comparison, we also use the 99.99% level (Lahaye *et al.* (2007)). Obviously the number of jumps decreases when we consider a higher confidence level. For instance, at the 99% and 99.9% confidence levels, 346 and 81 jumps are identified. At the highest confidence level, we can still identify 22 individual jumps and 20 jump days. These results are in line with those of Lahaye *et al.* (2007).²⁵ When the lowest confidence level is considered, there is quite a larger number of days in which more than one jump took place.

Period	Signif.	# jumps	# days with jumps	Max.	Min.	Mean	Std.
1	99%	119	80 (5.96%)	0.87%	-0.90%	-0.0346%	0.3491%
	99.9%	29	26 (1.94%)	0.87%	-0.90%	-0.0164%	0.4700%
	99.99%	7	5 (0.37%)	0.43%	-0.78%	-0.1552%	0.4854%
2	99%	84	51 (6.70%)	1.16%	-2.17%	-0.1079%	0.5522%
	99.9%	16	12 (1.58%)	0.45%	-0.97%	-0.3512%	0.4328%
	99.99%	3	3 (0.39%)	0.36%	-0.97%	-0.3412%	0.6655%
3	99%	143	80 (11.20%)	0.86%	-1.74%	0.0309%	0.3151%
	99.9%	36	31 (4.34%)	0.86%	-1.74%	-0.0284%	0.4513%
	99.99%	12	12 (1.68%)	0.63%	-1.74%	-0.2112%	0.6329%

Table 2: Descriptive statistics about jump occurrences. For each period and confidence level (“Signif.” in the table), the number of jumps and the number of days with jumps are reported. The two columns called “Max. SJ” and “Min. SJ” show the most positive and most negative returns due to a single jump. The last two columns, “Mean” and “Std.” report the mean and the standard deviation of all returns due to jumps.

On average, returns due to jumps are slightly negative and their standard deviations are large (between 50% and 120% in annualized terms, depending on the period under consideration and the significance level). There are some striking differences between the three periods. Notably, there are more days with jumps in the last period than in the other two. The second period was strongly bearish and highly volatile, while the last one was bullish and rather calm.²⁶ This difference seems therefore to be counterintuitive. Nevertheless, in a high volatility environment, relatively larger price changes are needed for the test statistics in (16) to become significant.

²⁵ Lahaye *et al.* (2007) study the number of jump occurrences for the Dow Jones, the Nasdaq and the S&P 500. They use data sampled each 15 minutes and consider a 99.99% confidence level. They find out that the proportion of jump days for these indices was respectively 1.43%, 0.7% and 1.70%.

²⁶ The average intradaily (i.e. without considering overnight returns) realized volatility was 15.22% in the first period, 25.04% in the second, and 14.94% in the last.

These results might indicate that the prices declined rather steadily during period two and that only a few returns were driven by sudden price jumps. Moreover, rare jumps were on average bigger during period two than during the other periods. When looking at the “Max.” and “Min.” columns of Table 2, one may also notice that the amplitude of the largest jumps tend to decrease when a higher confidence level is considered. This means that the largest price changes are not necessarily driven by jumps.

Table 3 shows the contribution of jumps to the overall daily variance. When considering all days (i.e. also non-jump days), their contribution remains quite restrained. But if we focus only on those days in which at least one jump takes place, the picture changes dramatically. In this case, about 15% to 40% of the daily variance is due to jumps. On some “extreme” days, more than 80% of the daily variance can be explained through jumps. It also seems that the impact of jumps on daily variance was bigger in the first period than in the next two periods.

Period	Signif.	All days	Jump days	Min.	Max.
1	99%	1.51%	25.38%	6.78%	79.47%
	99.9%	0.61%	31.62%	12.66%	80.82%
	99.99%	0.16%	42.24%	18.49%	71.34%
2	99%	1.13%	16.85%	4.89%	49.18%
	99.9%	0.22%	13.75%	4.66%	25.85%
	99.99%	0.06%	15.00%	10.82%	22.52%
3	99%	1.98%	17.70%	2.12%	50.00%
	99.9%	0.69%	15.93%	4.61%	47.78%
	99.99%	0.34%	20.31%	7.32%	49.11%

Table 3: Part of the daily realized variance due to jumps. For each period and confidence level (“Signif.” in the table), the average part of the daily variance that is due to jumps is reported (column “All days”). The last three columns shows the average (“Jump days”), minimal (“Min.”) and maximal (“Max.”) contribution of jumps to the daily variance for days in which at least a jump takes place.

5.2 Timing of jumps

In this subsection, we first analyze whether jumps occur randomly throughout the trading day. We then study the duration between two successive jumps.

DISTRIBUTION OF JUMPS THROUGHOUT THE TRADING DAY

We consider a typical trading day and study the probability for a jump to occur in each 5-minute interval, from the opening to the closing of the market. In most of these intervals, only a handful of jumps (or even no jump at all) have been identified. We therefore aggregate them into broader 20-minute intervals in order to get a more precise picture of their distribution throughout the day. We consider each 20-minute interval separately and calculate the proportion of days in

which a jump took place in that interval. The results are reported in figure 4. We focus on the 99% (left panel of the figure) and 99.9% (right panel) confidence levels as not enough jumps have been identified at the highest confidence level (99.99%) for conducting precise inferences.

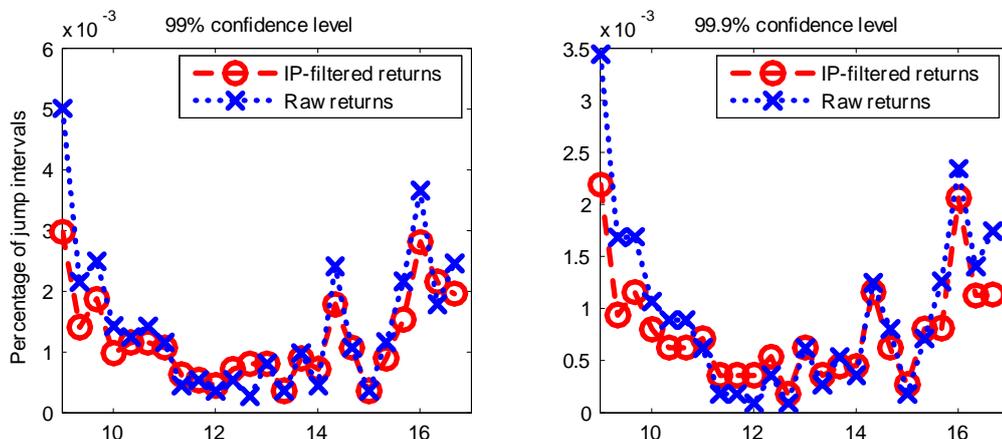


Figure 4: Distribution of jumps throughout the trading day. The numbers on the ordinate indicate the probability for a jump to occur in a 20-minute interval. The left and right panels are for the 99% and 99.9% confidence level.

Jumps are found in almost every interval but they do not spread evenly around the clock. Typically more jumps occur just after market opening, at about 2:30 p.m. and in the late afternoon. The raw and filtered series exhibit similar features. However, there are less jumps in the filtered series. This is because many large changes arise at particular moments of the day, when the market gets nervous. Thus many of them can be explained through the intraday patterns. This illustrates how crucial it is to filter out intraday patterns from raw returns before identifying jumps. All in one, these patterns are remarkably similar to the ones we get when studying intraday market activity (see section 4.1).

DURATION BETWEEN TWO SUCCESSIVE JUMPS

Table 4 shows the typical duration between two successive jumps for each period and each confidence level. In the first period and for the lowest confidence level, there was, on average, a jump each 15.79 (calendar) days. During the next two periods, jumps were more frequent and the average duration decreased to 7.06 days in the third period.

Period	Signif.	Mean	Std.	Min.	Max.
1	99%	15.79	22.62	0.0035	104.97
	99.9%	65.08	74.49	0.0208	349.90
	99.99%	251.01	340.83	0.0208	916.95
2	99%	13.07	18.50	0.0035	90.88
	99.9%	66.87	69.13	0.0243	210.87
	99.99%	322.07	438.28	12.1597	631.98
3	99%	7.06	11.26	0.0035	63.01
	99.9%	27.98	39.46	0.0069	196.68
	99.99%	78.73	69.07	4.0174	253.67

Table 4: Duration between two successive jumps. This table shows the duration between two jumps. For each confidence level and period, the average duration as well as the standard deviation, the minimum (Min.) and the maximum (Max.) of the duration are reported.

Figure 5 is complementary to this analysis as it shows the complete time-series of jumps for the 99% (upper panel in the figure) and 99.9% (lower panel) confidence levels. Sometimes there is more than one jump on the same day. For the lower confidence level, there are up to 12 jumps a day. This result looks a bit excessive. Indeed if we consider the 99.9% confidence level, the maximum amount of jumps a day decreases to 4. Furthermore, jumps tend to cluster. This is particularly striking when we focus on the 99.9% confidence level.²⁷

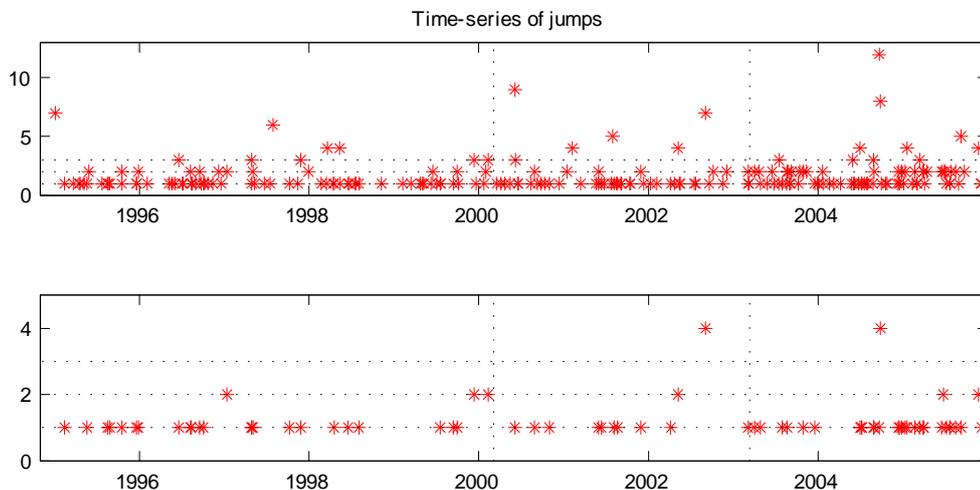


Figure 5: Time-series of jumps. The y-axis reports the number of jump occurrences on each day. The upper and lower panels are for the 99% and 99.9% confidence levels.

²⁷ Andersen *et al.* (2007) analyze jump occurrences for the S&P500 and find evidence of jumps clustering.

6 Impact of a jump on volatility

6.1 Impact on absolute returns and implied volatility

We follow the approach of Bollerslev *et al.* (2006) to examine the nature of the relationship between index returns and volatility. However, unlike Bollerslev *et al.* (2006), who use absolute returns as a proxy for volatility, we focus on implied volatility.

We calculate the correlation coefficient of DAX returns in a 5-minute interval j^* with implied volatility changes in 5-minute interval $j^* + k$, where $k \in \{-12, \dots, 12\}$. We denote by j^* the interval in which a jump has occurred. As a matter of comparison, we also examine the correlation between $r_{S,t_{j^*}}$ and the absolute return $|r_{S,t_{j^*+k}}|$.²⁸ The number of 12 leads and lags corresponds to 1 hour around the time at which the jump occurred. In order to quantify the additional impact of a jump on volatility, we also compute the correlation coefficients for the complete sample of returns (i.e. without making any distinction between jump and non-jump intervals). There is only a limited number of jumps in our sample. This might limit our ability to make precise inferences. Therefore, we make use of the complete 11 year sample and consider only jumps that have been identified at the 99% and 99.9% confidence levels.

Figure 6 reports the correlation coefficients between the different series. We first discuss the lead-lag effects among returns and absolute returns (left panel of the figure). Correlation coefficients for $k > 1$ and $k < 0$ are rather erratic and mostly insignificant. Though, at least for $k = 0$ and $k = 1$ they are consistently negative (about -0.2 to -0.4 for the 99% and 99.9% confidence levels) and significantly different from zero. By comparing these results with the ones we get for the complete sample (upper panel), we see that a jump has a real impact on volatility. Furthermore, this impact remains significant in the next 5-minute interval.

The right panels of the figure show the lead-effects for the implied volatility returns. These patterns are much clearer than those for the absolute returns. The correlation coefficients are significantly different from zero even for $k < 0$. This means that lagged volatility returns have a predictive power on future jump occurrences. We also find a significant and highly negative correlation of DAX returns with contemporaneous volatility returns ($k = 0$). At the 99% confidence level, the correlation coefficient is even more negative at $k = 1$ than at $k = 0$ and remains significant up to $k = 5$. This shows that the impact of a jump on volatility is important and also indicates that the market needs some time to fully incorporate the effects of the jump in the volatility process. At the 99.9% confidence level, the retarded reaction is even more pronounced as the correlation remains lower than -0.5 up to $k = 2$. If we compare these figures with the ones from the right upper panel, we see that the results mostly indicate the same kind of causality, running from index returns to volatility. Though the patterns are stronger in the case of jumps. This was expected because a large return caused by a jump should have more economic content than an innovation, which is only slightly different from zero. It is difficult to interpret the few significant correlations for $k < 0$. They seem to indicate that the market has some ability to predict a jump and its sign. If it is the case, we would expect to find

²⁸ This measure is closely related to the “skew” correlations of Engle & Lee (1993).

a positive autocorrelation between the returns recorded before the jump occurred and the jump return. We check this, but do not find any significant relationship.

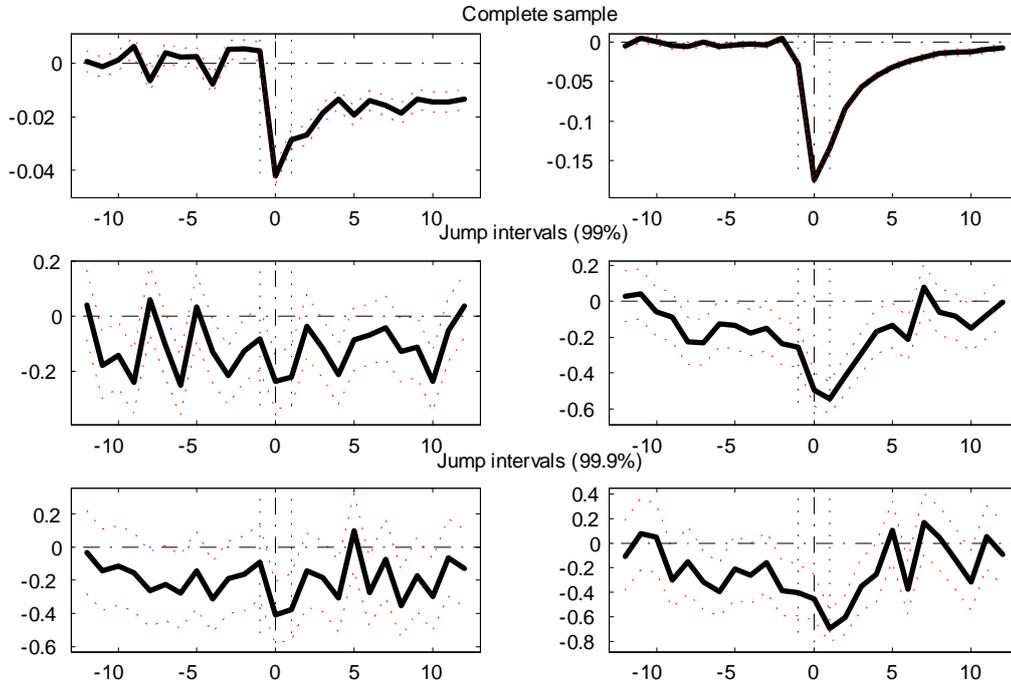


Figure 6: Lead-lag effects among DAX returns and absolute DAX returns (left panel) / returns on implied volatility (right panel). The upper panels report the results for the complete sample (i.e. without any distinction between jump and non-jump intervals), while the lower panels report the results for the jump intervals, where jumps are identified at the 99% and 99.9% confidence levels. The dotted lines correspond to the lower and upper bounds for a 95% confidence interval for each coefficient.

In order to gain further insight into this issue, we differentiate lead-lag effects for positive and negative jumps. This distinction obviously reduces the number of observations we have. Therefore we consider only jumps identified at the 99% confidence level and focus on a shorter time horizon of 30 minutes (i.e. $k \in \{-6, \dots, 6\}$). The correlation coefficients are reported in figure 7. The figure provides some evidence in favor of an asymmetric relationship. On the one hand, positive jumps do not seem to have much impact on volatility. On the other hand, negative jumps have an important effect on volatility. We also find some sort of volatility feed-back, as the correlation coefficients for $k = -2$ and $k = -1$ are significant. However, the causality remains mostly return-driven.²⁹

²⁹ We run another test in order to see if a jump might have an impact not only on the implied volatility level but also on the whole implied volatility smile structure. The results indicate that a jump does not have a significant effect neither on the slope nor on the curvature of the smile function.

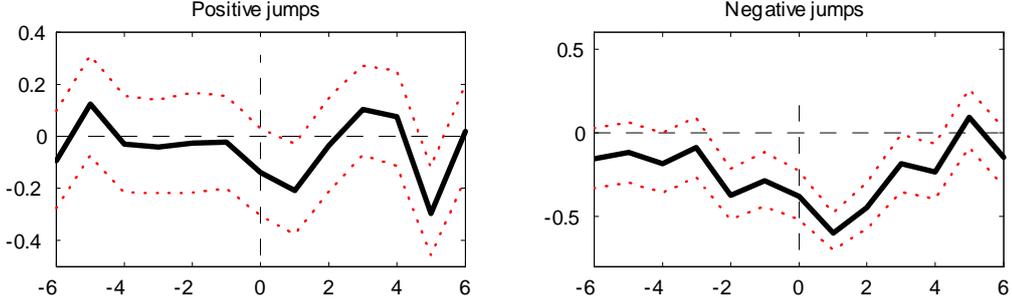


Figure 7: Lead-lag effects among positive (and negative, right panel) jump returns and returns on implied volatility. The dotted lines correspond to the lower and upper bounds for a 95% confidence interval for each coefficient.

6.2 Impact on realized volatility

In this section, we focus on the impact of a jump on the realized volatility. We first estimate the realized variance before and after a jump occurred:

$$RV_{K,t_j < t_j^*} = \sum_{k=1}^K r_{t_j^* - k}^2 \quad \text{and} \quad RV_{K,t_j > t_j^*} = \sum_{k=1}^K r_{t_j^* + k}^2,$$

where $RV_{K,t_j < t_j^*}$ and $RV_{K,t_j > t_j^*}$ are respectively the realized variance before and after a jump took place. Both measures are estimated over a horizon of K 5-minute intervals. t_j^* is the time at which a jump occurred. We consider various horizons, ranging from 5 minutes ($K = 1$) to 1 hour ($K = 12$). In order to render these measures more tractable, we reexpress them in terms of annualized volatility, i.e. $Rv = \sqrt{252 \cdot (100/K) \cdot RV}$ (as we have on average 100 5-minute intervals per day).

We then study the impact of a jump on the realized volatility over an horizon of K periods by taking the difference between $Rv_{K,t_j > t_j^*}$ and $Rv_{K,t_j < t_j^*}$. We differentiate between positive and negative jumps and get the following expressions:

$$\Delta Rv_K^+ = Rv_{K,t_j > t_{j+}^*} - Rv_{K,t_j < t_{j+}^*} \quad \text{and} \quad \Delta Rv_K^- = Rv_{K,t_j > t_{j-}^*} - Rv_{K,t_j < t_{j-}^*},$$

where ΔRv_K^+ and ΔRv_K^- respectively quantify the impact of a positive and a negative jump on realized volatility, t_{j+}^* (t_{j-}^*) is the time at which a positive (negative) jump has occurred.

Figure 8 shows that the realized volatility tends to increase after a negative jump has occurred. On an annualized basis, the increase amounts to about 49% for jumps identified at the 99% confidence level. The impact of the jumps identified at the 99.9% confidence level is even stronger (up to 62% increase in the realized volatility). However, this result is no longer significant at the 95% level. This could be due to the scarcity of significant jump observations. Positive jumps do not have a similar impact on the realized volatility. The most significant positive jumps seem to trigger a decrease in the realized volatility, while less significant jumps seem to have an opposite impact on the realized volatility. Yet, in both cases, the results are far from being significant at the standard levels.

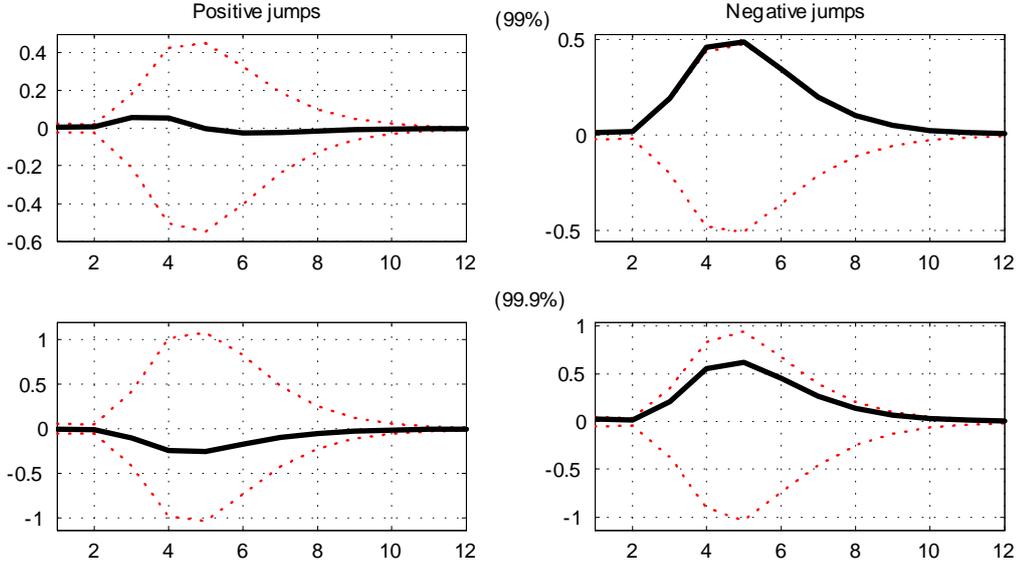


Figure 8: Impact of a jump on realized volatility over a horizon of K periods. The left (right) panel reports the results for positive (negative) jumps. The dotted lines show the upper and lower 95% confidence bounds.

7 Conclusion

In this paper, we examine three related empirical questions. First, we characterize the intraday patterns for both squared returns and implied volatilities. Second, we identify price discontinuities and study their statistical properties. Third, we analyze the impact of a price jump on the subsequent volatility level.

Our results indicate that intraday patterns for squared returns are typically W -shaped and have remained very stable over the sample period (1995-2005). The intraday volatility is particularly high at the market's opening, at 2:30 p.m. and at the closing. We also observe very clear-cut J -shaped patterns for implied volatility. These are more moderate than those for squared returns but, as indicated by a small empirical experiment (see section 4.2), their economic implications could be potentially important.

We identify some rare jumps. They occur on 0.37% and 11.20% of all days, depending on the period under consideration and the confidence level used to test their significance. The impact of jumps on the daily volatility is far from being negligible. On jump days, price discontinuities account on average for 15% to 25% of the daily return variance and, on some particular days, they explain as much as up to 80% of the daily return variance. We also notice that jumps tend to cluster. The length of our sample allows studying precisely the occurrences of jumps throughout the trading day. We find that they are distributed according to patterns that are very similar to the ones for squared returns. That is, there is a higher probability for a jump to occur in the early morning, at 2:30 p.m. and in the late afternoon.

Finally we find that price discontinuities have a considerable impact on volatility. A jump leads to a direct adjustment of the implied volatility level. This adjustment is upward or downward depending on whether the jump is negative or positive. Negative jumps have a much stronger effect on volatility than positive ones. The information conveyed by a jump needs some time to be fully integrated in the volatility process. That is, the impact of a jump on volatility remains significant during up to 25 minutes. Moreover, the subsequent impact of a jump on volatility tends to be even more important than its direct impact. We find almost similar results for absolute returns and realized volatility but these results are less significant. To sum up, we document (i) a direct impact of a jump on volatility; (ii) a retarded (or causal) effect of a jump on subsequent volatilities; (iii) a notable asymmetry in the relationship among jump returns and volatility changes.

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