

A High-Frequency Investigation of the Interaction between Volatility Changes and DAX Returns

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Abstract

One of the most noticeable stylized facts in finance is that stock index returns are negatively correlated with changes in volatility. The economic rationale for the effect is still controversial. The competing explanations have different implications for the origin of the relationship: Are volatility changes induced by index movements, or inversely, does volatility drive index returns? To differentiate between the alternative hypotheses, we analyze the lead-lag relationship of option implied volatility and index return in Germany based on Granger causality tests and impulse-response functions. Our dataset consists of all transactions in DAX options and futures over the time period from 1995 to 2005. Analyzing returns over 5-minute intervals, we find that the relationship is return-driven in the sense that index returns Granger cause volatility changes. This causal relationship is statistically and economically significant and can be clearly separated from the contemporaneous correlation. The largest part of the implied volatility response occurs immediately, but we also observe a smaller retarded reaction for up to one hour. A volatility feedback effect is not discernible. If it exists, the stock market appears to correctly anticipate its importance for index returns.

JEL classification: G10; G12; G13

Keywords: Implied volatility; Granger causality; leverage effect; feed-back effect; asymmetric volatility.

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1 Introduction

A well known stylized fact in finance is that stock index returns are negatively correlated with changes in volatility (Black (1976)). The negative relationship is typically more pronounced in falling than in rising markets (Figlewski/Wang (2000)) and is stronger for indices than for individual stocks. The distinctive cross dependence pattern between return and volatility plays an essential role in the development of volatility as an asset class, in volatility modelling and in option pricing. Nevertheless, a fully consistent economic explanation for the effect has not yet been offered (see Bouchaud et al. (2001), Bollerslev/Zhou (2006)).

The first attempt to find an economic rationale for the negative return correlation relies on a corporate finance argument. Black (1976) and Christie (1982), among others, argue that a positive stock return increases the market value of the firm's equity, thereby diminishing its financial leverage ratio. The reduced leverage gear will result in a lower volatility of stock returns. The empirical observations, however, do not support this leverage hypothesis. Firstly, it is not compatible with the observed asymmetry of the effect in falling and rising markets. Secondly, the leverage hypothesis predicts to find a stronger relationship on the individual firm level than the index level. This prediction is contrary to what is empirically observed (Bouchaud et al. (2001)).¹ In a US study, Figlewski/Wang (2000) conclude that the negative correlation on the index level is far too strong to be explained by the leverage hypothesis (see also Aydemir et al. (2006)).

The term "leverage effect" is sometimes used in a broader sense for the general hypothesis that the causality runs from stock return to volatility. In this paper, we call such a directional relationship "return-driven". In this terminology, the leverage effect is only one possibility to explain a return-driven negative correlation. Another explanation is that bad news might have different implications for future uncertainty than good news (see, e.g., Glosten et al. (1993) and Chen/Ghysels (2007)). For instance, price drops could induce more extensive portfolio adjustments of risk-averse agents than price increases. Bouchaud et al. (2001) suggest that the apparent return-driven relationship could be due to a retarded effect. In their framework, the scale for price updates does not depend on the instantaneous price but on a moving average of past prices which means that current returns lead subsequent volatility changes.

The hypothesis of a volatility-driven negative relationship is known as "feedback effect" (see, e.g., Pindyck (1984), French et al. (1987), Campbell/Hentschel (1992)). It rests on the assumption that volatility is related to systematic risk and is therefore relevant for pricing. If new information gives rise to an unanticipated increase in volatility, this will also increase risk-adjusted discount rates. As long as cash flow expectations are not affected, stock prices will go down. However, the empirical evidence on the impact of volatility on expected returns is controversial. Some studies report a positive (French et al. (1987), Campbell/Hentschel (1992), Scruggs (1998), Ghysels et al. (2005), Lundblad (2007), Bae et al. (2007)), others a negative relationship (Campbell (1987), Nelson (1991)). Often, the link was found to be insignificant and unstable over time (Glosten et al. (1993), Turner et al. (1989), Harvey (2001)).

¹ This statement holds although new evidence suggests that the firm level effect might be stronger than previous work has documented (see Ericsson et al. (2007), Chelley-Steeley/Steeley (2005)).

The return-driven and volatility-driven effects might well coexist. For instance, an initial price change could induce a volatility movement which in its turn amplifies the price change with yet another impulse on volatility (see the model of Bekaert/Wu (2000)). In efficient financial markets, the participants will try to anticipate these reactions. Therefore, the steps will evolve almost simultaneously. This makes it difficult to identify the different stages of the process. The higher the return frequency of the data, the better the chances to gain insight into the origin of the return-volatility correlation (see Bollerslev et al. (2006)).

Most empirical studies published during the last few years use a framework which incorporates return-driven as well as volatility-driven effects. The results are mixed. On the one hand, recent studies by Bollerslev et al. (2006), Giot (2005), and Dufour et al. (2006) report evidence of a return-driven relationship while the feedback effect is found to be negligible.² On the other hand, Bekaert/Wu (2000) and Dennis et al. (2006) find support for the volatility feedback argument.

To date, there is hardly any evidence for European countries. Within Europe, the German financial market appears to be particularly interesting for at least two reasons. Firstly, during our sample period, DAX futures and options have represented the highest trading volume among all stock index derivatives in Europe. Thus, high-quality high-frequency transaction data are available, which is of crucial importance for this study. Secondly, the negative relationship between index and volatility returns has been particularly strong and stable at the German market over the last decade (see Hafner/Wallmeier (2007)).

We analyze the lead-lag relationship between DAX returns and at-the-money (ATM) implied volatilities of DAX options over the 11-year period from 1995 to 2005. The time-stamped tick-by-tick data enable us to measure index and volatility returns over 5-minute intervals. In contrast to previous related work (Bollerslev et al. (2006), Dufour et al. (2006)), we study the ATM implied volatility instead of measures of realized volatility. This allows us to accurately determine the point in time when changes in volatility occur.³ Thus, the lead-lag relationship can directly be inferred from index and volatility returns over subsequent intervals. Based on these return data, we can go beyond a correlation analysis to investigate the causal relationship. Our objectives are: (1) to characterize the contemporaneous relationship between index and volatility returns taking asymmetry into account, (2) to analyze whether the lead-lag relationship is return-driven or volatility-driven, (3) to quantify the impact of an innovation in return or volatility and (4) to estimate how fast return-driven and feedback effects evolve and to draw conclusions for the information efficiency of the markets involved.

We find that the negative relationship between contemporaneous high-frequency index returns and volatility changes is almost linear and symmetric in rising and falling markets. A lead-lag relationship only exists for returns at the highest sampling frequency (5 minutes). This relationship is return-driven in the sense that index returns Granger cause volatility changes.

² Earlier studies include Christie (1982), Duffee (1995), Figlewski/Wang (2000) and Chan et al. (2003) (based on regressions) and Nelson (1991), Cheung/Ng (1992), Engle/Ng (1993) and Glosten et al. (1993) (based on modelling of conditional volatility through various GARCH specifications).

³ It is well known that ATM implied volatility is an upward-biased estimate of future realized volatility (see, e.g., Jackwerth/Rubinstein (1996)). We assume that the bias is approximately constant through time and therefore does not significantly influence volatility returns.

Impulse-response functions show that a one-time innovation on index return has a significant impact on implied volatility. The largest part of the implied volatility response occurs immediately, but we also observe a smaller retarded reaction for up to one hour. A volatility feedback effect is not discernible. If it exists, it appears to be correctly anticipated by traders in the stock market, so that the initial DAX return already incorporates the feedback from the expected volatility reaction.

In the next section, we describe our data and explain how we account for microstructure frictions. Sections 3 and 4 contain the correlation analysis and the causality tests, respectively. Section 5 concludes.

2 Data

2.1 Raw returns and the smile in option prices

Our data come from the joint German and Swiss options and futures exchange, Eurex.⁴ The Eurex is the world's largest futures and options exchange and is jointly operated by Deutsche Börse AG and SWX Swiss Exchange. Our database contains all reported transactions of options and futures on the German stock index DAX from January 1995 to December 2005. The average daily trading volume of DAX options (ODAX) and futures (FDAX) in December 2005 was 166,886 and 117,388 contracts. The options are European style. At any point in time during the sample period, at least eight option maturities were available. However, trading is heavily concentrated on the nearby maturities. Trading hours changed several times during our sample period, but both products were traded at least from 9:30 a.m. to 4:00 p.m.

To calculate the implied volatility for each transaction from the Black (1976) formula, it is crucial to accurately match the corresponding forward price. As we use time-stamped tick-by-tick data, matching of option and future prices is straightforward. We apply the method of Hafner/Wallmeier (2001) to ensure put-call-parity consistent estimates of implied volatilities. We remove all option prices which violate well-known arbitrage bounds as well as observations with an implied volatility larger than 150%.

Due to the smile in option prices, differences in implied volatilities of subsequent option prices can be due to different levels of moneyness defined as the quotient of strike price and forward price. To restrict the influence of the smile, we only keep at-the-money options with a moneyness between 0.975 and 1.025. Since a small influence of moneyness might still exist, we estimate the smile structure each day following the cubic regression approach described in Hafner/Wallmeier (2001) and Hafner/Wallmeier (2007). We then use the fitted smile function to remove the impact of moneyness on implied volatilities in the relevant moneyness range of 0.975 to 1.025. More specifically, let K denote the strike price of an option with time to maturity $T - t$. Each trade is assigned a moneyness according to:

$$M(t, T, K) = \frac{\ln\left(\frac{K}{F_t(T)}\right)}{\sqrt{T-t}},$$

⁴ We are very grateful to the Eurex for providing the data.

where $F_t(T)$ is the forward price at time t for maturity T . Thus, ATM options are characterized by a moneyness of 0. Suppressing the arguments of moneyness, we chose the cubic regression function:

$$\sigma = \beta_0 + \beta_1 M + \beta_2 M^2 + \beta_3 D \cdot M^3 + \varepsilon, \quad (1)$$

where σ is the implied volatility, β_i , $i = 0, 1, 2, 3$ are regression coefficients, ε is a random error, and D is a dummy variable defined as:

$$D = \begin{cases} 0 & , \quad M \leq 0 \\ 1 & , \quad M > 0 \end{cases} .$$

The dummy variable accounts for an asymmetry of the pattern of implied volatilities around the ATM strike ($M = 0$).

Let $\sigma_{imp}(M, t)$ denote the implied volatility of an option with moneyness M traded at time t . Then, the corresponding ATM implied volatility $\sigma_{imp}^{ATM}(t)$ is calculated as

$$\sigma_{imp}^{ATM}(t) = \sigma_{imp}(M, t) - [\widehat{\beta}_1 M + \widehat{\beta}_2 M^2 + \widehat{\beta}_3 D \cdot M^3],$$

where $\widehat{\beta}_i$ are the estimated regression coefficients.

We classify all observations into two maturity groups. The first contains options with a time-to-maturity between 10 and 30 calendar days, the second contains all observations with an option's time-to-maturity between 31 and 60 days. Options with longer maturities are not considered due to thin trading. Very short maturities below 10 days are also excluded to leave out expiration-day effects and to avoid biases due to inaccurate estimates of implied volatilities.

Relative changes of implied volatilities (R_v) and raw returns of the underlying stock index (R_S) over the time period from t_i to t_j are calculated as:

$$R_v(t_i, t_j) = \ln [\sigma_{imp}^{ATM}(t_j)] - \ln [\sigma_{imp}^{ATM}(t_i)] \quad \text{and} \quad R_S(t_i, t_j) = \ln [S(t_j)] - \ln [S(t_i)],$$

where S denotes the index level. The values $\sigma_{imp}^{ATM}(t)$ and $S(t)$ are set equal to the last implied volatility and index level observed before t . If the last trade occurred more than 60 seconds before t , the return is not calculated. The results do not change if we further restrict the maximal distance to 30 seconds. We consider four different sampling frequencies $t_j - t_i$, namely 5 minutes, 15 minutes, hourly and daily. We are primarily interested in the high-frequency 5-minutes intervals. Results for the longer intervals serve as a means of comparison.

2.2 Microstructure frictions

For data sampled at high frequency, market microstructure frictions have to be taken into account, particularly fluctuating trading activity, infrequent trading and the bid-ask bounce.

INTRADAY PATTERN OF TRADING ACTIVITY

Trading activity in DAX stocks and DAX options often follows an intraday pattern with three different periods: (1) in the morning, trading is typically at its maximum at the opening and

then decreases until lunch; (2) the period after lunch is generally characterized by a peak of trading activity around the opening of the US stock exchanges; (3) during the last few hours, activity usually stays at a high level. This intraday pattern may have consequences for the modelling of conditional volatility. For example, it can produce biases in GARCH specifications (see Andersen et al. (1999)). Due to the varying conditional volatility, returns in low-activity intervals are not directly comparable to returns in periods with high trading activity.

In order to correct DAX returns for the intraday trading activity, we follow Andersen et al. (2001) and model the pattern with a Fourier Flexible Form (*FFF*). The main assumption underlying this approach is that an intraday return can be expressed as the product of a Gaussian white noise, a daily (or long-term) volatility component and an intraday pattern effect. Using this decomposition, it is then possible to estimate and filter out the intraday pattern effect using a *FFF* regression (see Taylor (2006), Andersen et al. (2001) for more details).

Following Andersen et al. (2001), we only use the polynomial part of the *FFF* and break it up into three third order sub-polynomials to account for three different trading regimes during the day. Since the intraday pattern is not supposed to be constant over the eleven-year period, we treat each year separately. We apply this filtering only to return data sampled at the highest frequencies (5 and 15 minutes). While conceptually relevant, a robustness check shows that the filtering is not important empirically. Filtered returns turn out to be very similar to raw returns and all results remain valid when the filtering is omitted.

INFREQUENT TRADING

The problem of infrequent trading arises if intervals without market transactions occur. There are several ways to handle this problem. Some authors simply set the missing value equal to the last transaction price (e.g. Stephan/Whaley (1990) or Dennis et al. (2006)), while others interpolate between the last and the next price to fill the gap (see, e.g., Corsi et al. (2001) for a discussion). However, the first method has the shortcoming that it leads to a bias in vector autoregressions, whereas the second approach generates spurious autocorrelations. We therefore refrain from generating fictitious values to replace non-available market prices. Instead, in each part of the study we control for the presence of a sufficient number of available lags and discard data which do not satisfy this requirement.

BID-ASK BOUNCE

As is well known, the bid-ask bounce leads to a negative first order autocorrelation of returns (Roll (1984)). This spurious autocorrelation comes from successive trades, where one is executed at the bid, the other at the ask price. The bid-ask bounce effect is more pronounced for DAX options with their relatively large bid-ask spreads than for DAX futures and the underlying index. We use the standard method to remove spurious autocorrelation from returns, which consists in filtering the returns with a MA(1) process (see, e.g., Stephan/Whaley (1990), Easley et al. (1998) and Gwilym/Buckle (2001)):

$$R_t = \mu + e_t - \theta e_{t-1},$$

where R_t is the observed return, μ the unconditional mean of the return series, θ the moving average coefficient, and e_t the innovation of the process. Since the innovations from the MA(1) process are uncorrelated, we can use them as bid-ask bounce corrected returns. Through the FFF filtering and the MA(1) correction, raw returns R_v and R_S are transformed into adjusted r_v and r_S which are used in the empirical tests.

2.3 Descriptive return statistics

Descriptive statistics for DAX and volatility 5-minute log returns (raw returns R_S and R_v as well as adjusted returns r_S and r_v) are given in Table 1. The changes in implied volatilities are reported for a time to maturity of 31 to 60 days. Results for the shorter maturity are very similar. For better comparison, all statistics are given on a daily basis. We compute the summary statistics for different time-windows: each year, the whole 11-year period (All) and three subperiods corresponding to the long bullish market of the late nineties (P1: 01/01/1995 to 03/07/2000), the bearish market that followed the end of the tech bubble and the 9/11 attacks (P2: 03/08/2000 to 03/12/2003) and the bullish market that took place after the beginning of the Iraq War (P3: 03/13/2003 to 12/31/2005).

In most years, skewness of DAX returns is slightly negative and kurtosis widely exceeds three. P -values of the Jarque-Bera test (not reported in the table) unambiguously reject normality on the 1% level. The bearish market (P2) is characterized by a negative mean, high variance and kurtosis and strongly negative skewness. Volatility returns have a much higher variance than DAX returns. The sign of skewness of volatility return varies. Kurtosis typically exceeds three, so that the null of normality is rejected for all subsamples. The adjustments of raw returns to account for intraday patterns and microstructure effects have only marginal effects on DAX returns, whereas the MA(1) correction for variance returns noticeably modifies (unconditional) variance and skewness.

3 Correlation analysis

3.1 Contemporaneous correlation and asymmetry

The contemporaneous cross-correlation between index and volatility returns is always negative and highly significant (see Table 2). It is almost the same regardless of whether implied volatilities are computed from the nearest-to-maturity options or options with the following maturity. The strength of the correlation strongly depends on the return frequency: Daily return correlations vary between -0.6 and -0.8, whereas 5-minute returns have correlation coefficients of about -0.1 to -0.3. This difference is primarily due to noise in high-frequency returns.

Year	R_S (5-minute)			r_S (5-minute)			R_v (5-minute)			r_v (5-minute)						
	Mean	Std.	K.	Mean	Std.	K.	Mean	Std.	K.	Mean	Std.	K.				
1995	-0.0008	0.0066	-0.12	9.93	-0.0007	0.0065	-0.07	10.51	-0.0023	0.1690	0.50	9.98	-0.0057	0.1557	0.54	12.05
1996	0.0005	0.0058	-0.41	11.37	0.0004	0.0057	-0.57	14.13	0.0453	0.1853	-0.06	5.44	0.0408	0.1712	-0.07	5.05
1997	0.0001	0.0115	-0.53	18.65	0.0000	0.0111	-0.55	14.35	0.0158	0.2154	-0.22	16.00	0.0308	0.2006	-0.59	11.95
1998	-0.0007	0.0149	-0.24	9.66	-0.0005	0.0147	-0.21	9.22	0.0247	0.1537	-0.46	11.74	0.0190	0.1448	-0.74	11.05
1999	0.0004	0.0117	-0.21	7.39	0.0004	0.0116	-0.27	7.56	-0.0407	0.1399	-0.14	6.43	-0.0466	0.1328	-0.25	6.50
2000	-0.0005	0.0121	-0.18	6.79	-0.0004	0.0119	-0.20	6.18	-0.0116	0.1586	-0.40	10.77	-0.0119	0.1475	-0.68	8.01
2001	-0.0006	0.0172	-1.62	116.86	-0.0006	0.0175	-1.78	205.46	0.1044	0.2246	0.41	10.51	0.1191	0.2125	0.40	10.53
2002	-0.0018	0.0208	-0.02	8.76	-0.0016	0.0203	-0.11	7.11	0.0338	0.1826	0.29	11.05	0.0362	0.1733	0.39	11.91
2003	0.0003	0.0167	-0.05	7.61	0.0008	0.0164	-0.01	7.01	0.0145	0.1457	-0.23	9.54	0.0189	0.1392	-0.28	9.06
2004	-0.0005	0.0076	-0.07	8.94	-0.0004	0.0074	-0.04	11.20	0.0154	0.1191	0.28	8.95	0.0206	0.1137	0.36	8.44
2005	0.0004	0.0058	-1.10	34.57	0.0003	0.0057	-1.33	51.96	0.0145	0.1333	0.56	18.68	0.0237	0.1283	0.70	19.17
P1	0.0000	0.0111	-0.32	14.14	0.0001	0.0110	-0.31	12.69	0.0115	0.1762	-0.09	12.53	0.0098	0.1639	-0.29	10.48
P2	-0.0015	0.0180	-0.56	42.84	-0.0013	0.0178	-0.71	75.36	0.0398	0.1934	0.34	11.94	0.0475	0.1828	0.36	12.12
P3	0.0006	0.0103	0.03	14.88	0.0007	0.0101	0.11	13.86	0.0174	0.1307	0.29	14.19	0.0251	0.1255	0.34	14.39
All	-0.0003	0.0136	-0.53	53.10	-0.0002	0.0134	-0.65	91.86	0.0190	0.1673	0.15	13.89	0.0221	0.1572	0.08	13.07

Table 1: Summary statistics. We report Mean, Standard deviation (Std.), Skewness (S.) and Kurtosis (K.) for 5-minute stock index (DAX) returns and 5-minute implied volatility returns for options with the second nearest time-to-maturity. Mean and standard deviation are given on a daily basis. Periods P1 to P3 are 01/01/1995 - 03/07/2000 (P1), 03/08/2000 - 03/12/2003 (P2) and 03/13/2003 - 12/31/2005 (P3).

	10 Days < Time-to-Maturity \leq 1 Month						1 Month < Time-to-Maturity \leq 2 Months									
	$\rho(R_S, R_v)$			$\rho(r_S, r_v)$			$\rho(R_S, R_v)$			$\rho(r_S, r_v)$						
	5m.	15m.	1h.	1d.	5m.	15m.	1h.	1d.	5m.	15m.	1h.	1d.				
1995	-0.15	-0.25	-0.33	-0.55	-0.10	-0.29	-0.23	-0.59	-0.16	-0.26	-0.37	-0.56	-0.10	-0.31	-0.23	-0.59
1996	-0.14	-0.24	-0.62	-0.53	-0.18	-0.26	-0.40	-0.70	-0.15	-0.24	-0.61	-0.52	-0.19	-0.28	-0.40	-0.69
1997	-0.15	-0.30	-0.39	-0.60	-0.10	-0.27	-0.12	-0.65	-0.15	-0.29	-0.40	-0.60	-0.07	-0.26	-0.12	-0.65
1998	-0.25	-0.40	-0.56	-0.71	-0.21	-0.35	-0.41	-0.77	-0.26	-0.41	-0.56	-0.73	-0.21	-0.36	-0.41	-0.78
1999	-0.23	-0.36	-0.44	-0.51	-0.24	-0.36	-0.47	-0.55	-0.24	-0.36	-0.45	-0.52	-0.24	-0.33	-0.46	-0.56
2000	-0.16	-0.32	-0.60	-0.40	-0.12	-0.19	-0.46	-0.56	-0.17	-0.33	-0.59	-0.42	-0.11	-0.22	-0.46	-0.57
2001	-0.25	-0.29	-0.89	-0.68	-0.24	-0.28	-0.61	-0.77	-0.25	-0.31	-0.88	-0.70	-0.21	-0.29	-0.64	-0.78
2002	-0.20	-0.36	-0.49	-0.71	-0.21	-0.28	-0.64	-0.71	-0.23	-0.36	-0.48	-0.71	-0.20	-0.30	-0.64	-0.69
2003	-0.23	-0.37	-0.74	-0.50	-0.26	-0.40	-0.44	-0.55	-0.24	-0.37	-0.75	-0.50	-0.28	-0.36	-0.46	-0.55
2004	-0.24	-0.40	-0.65	-0.74	-0.26	-0.39	-0.66	-0.81	-0.27	-0.43	-0.64	-0.75	-0.26	-0.38	-0.65	-0.82
2005	-0.16	-0.26	-0.51	-0.61	-0.19	-0.40	-0.54	-0.66	-0.19	-0.28	-0.51	-0.61	-0.19	-0.42	-0.53	-0.67
P1	-0.18	-0.31	-0.50	-0.59	-0.15	-0.28	-0.32	-0.65	-0.19	-0.31	-0.50	-0.60	-0.14	-0.28	-0.32	-0.65
P2	-0.20	-0.31	-0.76	-0.64	-0.20	-0.28	-0.58	-0.69	-0.21	-0.32	-0.75	-0.64	-0.18	-0.29	-0.59	-0.69
P3	-0.20	-0.32	-0.56	-0.56	-0.22	-0.35	-0.53	-0.60	-0.22	-0.33	-0.56	-0.57	-0.22	-0.35	-0.53	-0.61
All	-0.18	-0.30	-0.60	-0.59	-0.17	-0.29	-0.43	-0.64	-0.20	-0.31	-0.60	-0.60	-0.17	-0.30	-0.44	-0.65

Table 2: Cross-correlations between stock index returns and implied volatility returns. Options with the first and second shortest time-to-maturity are taken into account. Results are given for raw and adjusted returns and four sampling frequencies, 5-minute (5m. in the table), 15-minute (15m.), hourly (1h.) and daily (1d.). Correlations are computed for each year (1995 - 2005), for three subperiods (P1 - P3) and for the whole sample (All). The subperiods are 01/01/1995 - 03/07/2000 (P1), 03/08/2000 - 03/12/2003 (P2) and 03/13/2003 - 12/31/2005 (P3). All coefficients are significant at the 99%-level.

		10 Days < Time-to-Maturity \leq 1 Month											
		01/01/1995 - 03/07/2000				03/08/2000 to 03/12/2003				03/13/2003 to 12/31/2005			
		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	R^2
5 m.		0.0002	-2.5396	-0.3358	0.0367	-0.0001	-1.4945	-0.8023	0.0455	0.0000	-3.3025	-0.1885	0.0475
		1.41	-13.24	-1.12	<0.0001	-0.34	-4.17	-1.16	<0.0001	0.26	-13.07	-0.47	<0.0001
30 m.		0.0003	-2.5712	-0.5047	0.0983	-0.0003	-1.3207	-1.0671	0.1066	0.0000	-3.3778	-0.4338	0.1117
		0.95	-9.15	-1.16	<0.0001	-0.81	-4.07	-2.31	<0.0001	0.11	-9.94	-0.83	<0.0001
1 h.		0.0027	-3.2779	-0.7212	0.2517	-0.0091	-0.7539	-4.4509	0.6330	0.0021	-4.7238	1.2563	0.3171
		1.93	-5.89	-0.85	<0.0001	-2.69	-1.00	-3.00	<0.0001	1.62	-7.09	1.32	<0.0001
1 day		-0.0092	-1.7003	-2.5343	0.3951	-0.0047	-1.9081	-0.1083	0.4140	-0.0090	-1.5541	-2.2378	0.3572
		-3.17	-4.60	-4.55	<0.0001	-0.98	-4.10	-0.17	<0.0001	-1.67	-2.08	-1.82	<0.0001
		1Month < Time-to-Maturity \leq 2 Months											
		01/01/1995 - 03/07/2000				03/08/2000 to 03/12/2003				03/13/2003 to 12/31/2005			
		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	R^2
5 m.		0.0002	-2.2539	0.3246	0.0209	0.0001	-1.6144	-0.3572	0.0343	-0.0001	-2.4509	-0.7753	0.0510
		0.90	-6.68	0.77	<0.0001	0.13	-4.77	-0.53	<0.0001	-0.42	-6.75	-1.63	<0.0001
30 m.		0.0006	-2.8589	0.3816	0.0802	0.0001	-1.6113	-0.4539	0.0871	-0.0006	-2.3561	-1.3830	0.1254
		1.24	-6.19	0.59	<0.0001	0.09	-5.19	-0.68	<0.0001	-1.08	-5.18	-1.33	<0.0001
1 h.		0.0016	-1.8169	-0.4797	0.1038	-0.0031	-1.4533	-0.9813	0.3564	-0.0014	-2.7460	-1.0119	0.2798
		1.66	-5.15	-0.82	<0.0001	-1.98	-4.39	-1.60	<0.0001	-1.19	-4.46	-1.12	<0.0001
1 day		-0.0064	-1.5686	-1.8539	0.4526	-0.0102	-1.3358	-1.0926	0.4872	-0.0066	-1.5544	-1.7801	0.3921
		-4.04	-7.79	-5.61	<0.0001	-3.16	-7.13	-2.42	<0.0001	-2.66	-4.56	-2.66	<0.0001

Table 3: Results for the asymmetric regression. For each sampling frequency (first column), the coefficient estimates and the R^2 of the regression are reported, with t -stats (for coefficients) and p -values (for R^2) given below.

To investigate if the return correlation is asymmetric, we regress volatility changes against both positive and negative DAX returns according to the following specification:⁵

$$r_{v,t} = \alpha + \beta r_{S,t} + \gamma r_{S,t} \cdot I_{\{r_{S,t} < 0\}} + \varepsilon_t, \quad (2)$$

where α , β , γ are the regression coefficients and $I_{\{r_{S,t} < 0\}}$ is an indicator variable that takes on the value 1 if the DAX return is negative and 0 otherwise. Thus, β reflects the slope coefficient for positive DAX returns, and $(\beta + \gamma)$ the corresponding coefficient for negative index returns. An asymmetry is validated if the γ estimates are significantly different from zero. We expect $\hat{\gamma}$ to be significantly negative because previous literature documented that the so-called leverage effect mainly consists of a down-market effect (Figlewski/Wang (2000)). A residual diagnosis did not reveal systematic deviations from the linear model of equation (2). In particular, including the index return squared as an explanatory variable does not improve the explanatory power of the regression model.

In Table 3, we report coefficient estimates and their t -statistics based on Newey/West (1987) adjusted standard errors. Naturally, the R^2 shows the same dependence on the return frequencies as the correlation coefficients reported earlier. As expected, all β estimates are significantly negative. They tend to be the more negative the higher the return frequency. Therefore, a positive DAX return in a 5-minute interval is typically combined with a larger relative decrease in volatility than a daily DAX return of equal size. The asymmetry coefficient $\hat{\gamma}$ is mostly negative, but low and insignificant for high-frequency returns. In contrast, for daily returns, $\hat{\gamma}$ is significantly negative in five out of six combinations of option maturity and sub-period. Thus, an asymmetry seems to exist only at the level of low-frequency (daily) returns. This observation is not compatible with a constant, time-independent relationship between high-frequency DAX and volatility returns. A stream of literature has examined why financial variables might exhibit this kind of heterogeneity⁶. Discontinuities and long memory of the latent volatility process have been put forward as possible explanations (see, e.g., Oswiecimka et al. (2005)).

3.2 Correlation with lagged returns

To examine the lead-lag relationship between index and volatility returns, we calculate the correlation coefficient of DAX returns in a 5-minute interval t with implied volatility changes in 5-minute interval $t + j$, where $j \in \{-250, \dots, 250\}$. As one trading day typically comprises about 100 intervals of 5 minutes, the number of 250 leads and lags corresponds to 5 trading days around t . Calculations are based on all t during the total sample period. Figure 1 shows that the correlation coefficient is near zero for lagged volatility ($j < 0$). Thus, the DAX return does not seem to be systematically related to the preceding change in implied volatility. However, we find a significantly negative correlation of DAX returns not only with contemporaneous volatility

⁵ The term ‘‘asymmetric volatility’’ is sometimes used as a short form for the observation that volatility reacts asymmetrically to index returns: it increases when index returns are positive and decreases when they are negative. In contrast, our definition of asymmetry is more specific in that it refers to the relative magnitude of volatility increases and decreases, given that an ‘‘asymmetric volatility’’ in the more general sense exists.

⁶ See the discussion about the mono- or multifractal nature of financial returns in Gençay et al. (2005), Gençay/Selçuk (2006) and Mandelbrot/Hudson (2006), among others.

returns ($j = 0$), but also with volatility returns in the next few 5-minute periods. About 1 hour after the stock price shock ($j > 12$), the correlation goes back to zero. This observation supports the hypothesis that implied volatility is adjusted to changes in the index level, so that the relationship seems to be primarily return-driven. It is important to note that the retarded reaction of volatility cannot be explained by thin trading and missing volatility returns, because the return in period $t + j$ is calculated only if transaction prices at the beginning and the end of the period are available. If the return in $t + 2$ is available while the $t + 1$ -return is missing, the implied volatility at the beginning of period $t + 2$ (which is available) should already reflect the price innovation in t . Thus, the return in $t + 2$ should not be influenced regardless of whether the $t + 1$ -return is available or not.

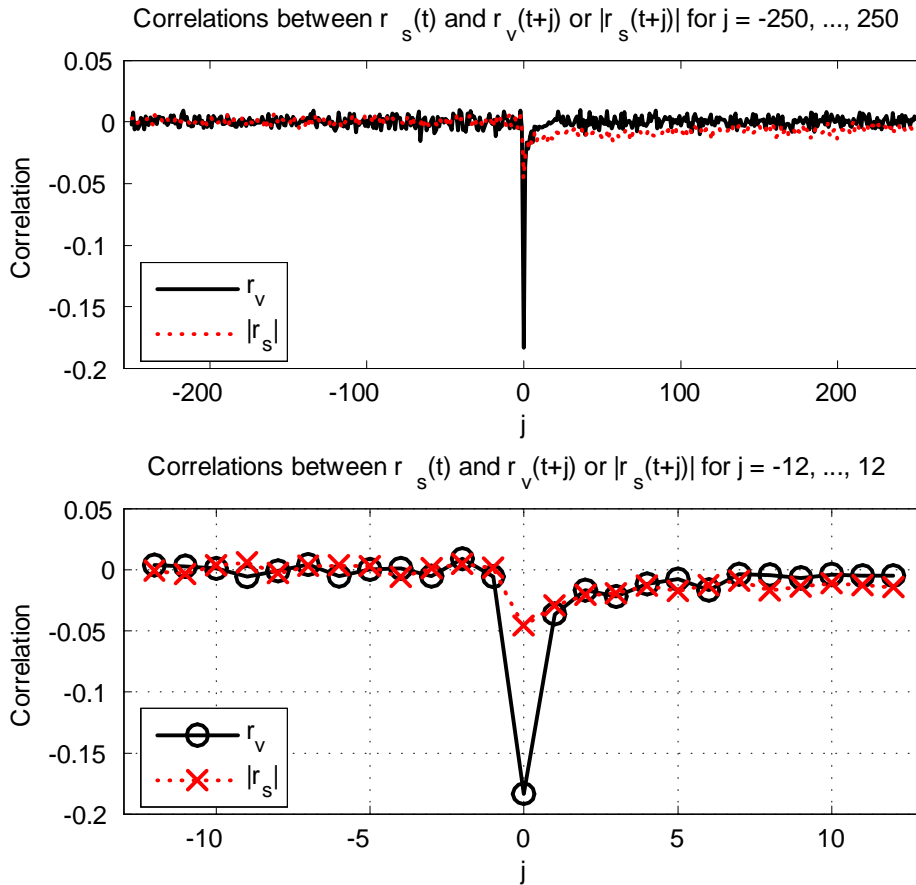


Figure 1: Correlations for the first maturity and 5-minute returns over the total sample period from 1995 to 2005. The upper panel reports correlations for $j = -250, \dots, 250$ while the bottom panel focuses on correlations for $j = -12, \dots, 12$.

Following Bollerslev et al. (2006), we also examine the correlation between $r_{S,t}$ and the absolute return $|r_{S,t+j}|$. In this specification, absolute returns serve as an alternative measure of realized volatility. It is apparent from Figure 1 that there is no noticeable correlation of absolute returns before t with DAX return in t . The contemporaneous correlation in t is negative, which means that negative returns are typically larger in magnitude than positive returns. After t , absolute returns are negatively correlated with $r_{S,t}$. This relationship gets weaker the larger the lag, but

it is recognizable for all $j \in \{1, \dots, 250\}$. These observations are similar to the US results in the study of Bollerslev et al. (2006). The correlation series for absolute returns shows that negative DAX returns typically increase subsequent return dispersion. Again, this is compatible with a return-driven effect. However, the analysis focuses on total cross-autocorrelations only and leaves out partial cross-autocorrelations. It could be the case that correlations computed for lags $j \geq 2$ are completely due to the correlation at lag $j = 1$. A more detailed study is thus necessary to identify causality and the number of lagged DAX returns which have an impact on contemporaneous volatility returns.

4 Causality analysis

4.1 Granger causality test

We carry out a Granger causality test, i.e. each variable is regressed on a constant and p of its own lags as well as on p lags of the other variable in terms of the following VAR(p) vector autoregression:

$$\mathbf{R}_t = \mathbf{c} + \sum_{i=1}^p \mathbf{\Phi}^{(i)} \cdot \mathbf{R}_{t-i} + \boldsymbol{\varepsilon}_t, \quad (3)$$

where \mathbf{R}_t is the (2×1) vector of DAX and volatility returns, \mathbf{c} is the (2×1) vector of constants and $\mathbf{\Phi}^{(i)}$ is the (2×2) matrix of autoregressive slope coefficients for lag i . The two equations of the vector system (3) specify that:

$$r_{S,t} = c_1 + \sum_{i=1}^p \phi_{11}^{(i)} \cdot r_{S,t-i} + \sum_{i=1}^p \phi_{12}^{(i)} \cdot r_{v,t-i} + \varepsilon_{1,t}, \quad (4)$$

$$r_{v,t} = c_2 + \sum_{i=1}^p \phi_{21}^{(i)} \cdot r_{S,t-i} + \sum_{i=1}^p \phi_{22}^{(i)} \cdot r_{v,t-i} + \varepsilon_{2,t}, \quad (5)$$

where c_i denotes the i th element of the vector \mathbf{c} and $\phi_{jk}^{(i)}$ denotes the row j , column k element of the matrix $\mathbf{\Phi}^{(i)}$. We call (4) the *returns regression (RR)* and (5) the *volatility changes regression (VCR)*. We estimate both regressions separately employing Ordinary Least Squares (OLS).⁷ In order to account for heteroskedasticity of the errors $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, we calculate standard errors according to the Newey/West (1987) procedure.

If for all $s > 0$ the mean squared error (MSE) of a forecast of $r_{S,t+s}$ based on $(r_{S,t}, \dots, r_{S,t-p})$ is the same as the MSE of a forecast of $r_{S,t+s}$ based both on $(r_{S,t}, \dots, r_{S,t-p})$ and $(r_{v,t}, \dots, r_{v,t-p})$, we conclude that $r_{v,t}$ fails to *Granger-cause* $r_{S,t}$ (see, e.g., Hamilton (1994)). Hence testing if any of the variables leads the other requires testing whether the cross-coefficients $\phi_{12}^{(i)}$, $\phi_{21}^{(i)}$ ($i = 1, \dots, p$) are different from zero. Formally, the validity of the return-driven relationship implies that all matrices $\mathbf{\Phi}^{(i)}$, $i = 1, \dots, p$ are lower triangular. In the same vein, the validity of the volatility-driven relationship requires all matrices $\mathbf{\Phi}^{(i)}$, $i = 1, \dots, p$ to be upper triangular. In the case where all matrices $\mathbf{\Phi}^{(i)}$, $i = 1, \dots, p$ are diagonal, both explanations have to be

⁷ For a covariance-stationary process, this estimation is consistent, see Hamilton (1994).

rejected. The two explanations could also cohabit, which is the case if some matrices $\Phi^{(i)}$ are full. To test which of the different cases is the most realistic, we estimate a series of unrestricted and restricted regressions and compare their explanatory power. The unrestricted regression makes use of the full set of variables ($r_{S,t-i}$ and $r_{v,t-i}$) to explain $r_{S,t}$ (RR) and $r_{v,t}$ (VCR), while the restricted version only uses lagged values of the regressand as explanatory variables.

Table 4 summarizes the results of the Wald F -test for the eleven-year sample period and each sampling frequency for up to five lags ($p = 5$).⁸ The F -statistics is used to test whether the return-driven or volatility driven relationship can be rejected. In each case, we report the p -value of the F -test (first line) and the number of observations included in the regression (second line). When more lags are considered, the number of complete return series and therefore the number of observations sharply decreases.⁹ This is why we focus our attention primarily on the case of $p = 1$. We regard the rejection of an effect as more reliable if the p -values for both option maturities are significant.

The results provide evidence in favor of a return-driven relationship. At the highest sampling frequency, the null hypothesis that past returns do not contribute to the explanation of current changes in implied volatility is always rejected at least at the 1% significance level. This causality is discernible for intervals of up to 60 minutes. The same results hold true in yearly subsamples except for hourly data. The return-driven relationship for hourly data only comes from the subperiod 03/08/2000 to 03/12/2003. The test statistics do not give evidence for a volatility-driven effect. Past volatility changes do not significantly add to the explanatory power of past index returns in explaining current index returns¹⁰. Results for periodical subsamples are similar to those for the total sample period.¹¹ The only notable difference is that we find a causality running in both directions in the bearish market from 2000 to 2003 when considering volatility computed from options with the second nearest time-to-maturity. In fact, the significant daily return-driven relationship that we find for the total sample comes entirely from this period. In all, we conclude that the lead-lag relationship of DAX returns and implied volatility changes is compatible with Granger causality running from return to volatility.

The results achieved so far do not rule out the possibility that a feedback-effect was not detected because it occurs in a more subtle fashion. For instance, one may suspect that only large volatility changes have an impact on returns. To investigate if such non-linear feedback-effects exist, we performed a non-parametric causality test introduced by Baek/Brock (1992) and extended and improved by Hiemstra/Jones (1994) and Diks/Panchenko (2005). This test examines if the probability distribution of future index returns is different if the information set contains either the history of both DAX and volatility returns or the history of DAX returns alone. The test

⁸ According to the Akaike and Schwartz information criteria, the optimal number of lags varies between 2 and 5 for sampling frequencies of 5 and 15 minutes and is equal to 1 for hourly and daily data.

⁹ The number of hourly data is relatively low, because we calculate hourly returns only if a transaction price is available from the last 60 seconds (see Section 2).

¹⁰ There is weak evidence in favor of a volatility-driven effect when considering 5-minute returns. However, this result depends on the number of lags and the time-to-maturity. Therefore, its economic significance seems questionable.

¹¹ The results for the three subsamples P1 to P3 are available on request.

		10 Days < Time-to-Maturity \leq 1 Month				
		Return-driven relationship with $p = \dots$				
		1	2	3	4	5
5m.	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
	39137	27434	19930	14858	11302	11302
15m.	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
	11138	7120	4671	3099	2073	2073
1h.	.0012	.0021	.0319	.4499	.7632	.6831
	598	281	133	70	39	39
1d.	.6335	.3989	.4981	.6869	.5154	.6044
	1591	1458	1325	1193	1061	1061
		1 Month < Time-to-Maturity \leq 2 Months				
		Return-driven relationship with $p = \dots$				
		1	2	3	4	5
5m.	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
	12070	6642	4086	2706	1854	1854
15m.	<.0001	<.0001	<.0001	.0005	.1135	.5702
	3272	1668	920	539	324	324
1h.	.0002	<.0001	.0187	.1102	.7569	.2540
	746	265	116	54	24	24
1d.	.0130	.0532	.0219	.0491	.0137	.1178
	2774	2773	2772	2771	2770	2770
		Volatility-driven relationship with $p = \dots$				
		1	2	3	4	5
5m.	<.0001	.1143	.0536	.0128	.0019	.0069
	39137	27434	19930	14858	11302	11302
15m.	<.0001	.0735	.8248	.3546	.8687	.9148
	11138	7120	4671	3099	2073	2073
1h.	.4472	.2405	.5287	.2835	.6831	.6831
	598	281	133	70	39	39
1d.	.9092	.2530	.3598	.5171	.6044	.6044
	1591	1458	1325	1193	1061	1061
		Volatility-driven relationship with $p = \dots$				
		1	2	3	4	5
5m.	<.0001	.0887	.7341	.0073	.1023	.4523
	12070	6642	4086	2706	1854	1854
15m.	<.0001	.1026	.7916	.6121	.7598	.5702
	3272	1668	920	539	324	324
1h.	.7663	.8132	.0989	.0704	.2540	.2540
	746	265	116	54	24	24
1d.	.0594	.1783	.1621	.1446	.1178	.1178
	2774	2773	2772	2771	2770	2770

Table 4: Granger causality test over the total sample period from 1995 to 2005. We report p -values of the F -tests for up to 5 lags in the first line and the number of available data (i.e. of successive returns) in the second line. Sampling frequencies are 5 minutes (5m.), 15 minutes (15m.), hourly (1h.) and daily (1d.).

statistics of the test by Diks/Panchenko (2005) (not shown here) do not provide evidence in favor of a non-linear feedback effect.

4.2 Contemporaneous versus lagged relationship

The finding of a return-driven effect in high-frequency data leaves open the question of how important this lead-lag-relationship is compared to the strong contemporaneous correlation of index and volatility return found in Section 3. To enable this comparison, we extend the volatility changes regression (5) by adding contemporaneous DAX returns as explanatory variable:

$$r_{v,t} = c^* + \sum_{i=1}^p \phi_{i,1}^* \cdot r_{S,t-i} + \sum_{i=1}^p \phi_{i,2}^* \cdot r_{v,t-i} + \beta^* r_{S,t} + \varepsilon_t^*. \quad (6)$$

We compare the unrestricted model (6) with two restricted versions:

- restricted model 1, characterized by $\beta^* = 0$, and
- restricted model 2, characterized by $\phi_{i,1}^* = 0 \quad \forall i = 1, \dots, p$.

Restricted model 1 is identical to the volatility changes regression of the last section, whereas restricted model 2 replaces lagged index returns by the contemporaneous index return as explanatory variable. Using OLS with Newey/West (1987) standard errors, we estimate the three regressions for all sampling frequencies and the two times-to-maturity. We also decompose the variance V of volatility changes according to:

$$\begin{aligned} V(r_{v,t}) &= V\left(\sum_{i=1}^p \phi_{i,1}^* \cdot r_{S,t-i} + \sum_{i=1}^p \phi_{i,2}^* \cdot r_{v,t-i} + \beta^* r_{S,t} + \varepsilon_t^*\right) \\ &= V(r_{v,t}) \cdot (VL + VCR + COV + VE), \end{aligned} \quad (7)$$

where

$$\begin{aligned} VL &= V\left(\sum_{i=1}^p \phi_{i,1}^* \cdot r_{S,t-i} + \sum_{i=1}^p \phi_{i,2}^* \cdot r_{v,t-i}\right) / V(r_{v,t}), \\ VCR &= V(\beta^* r_{S,t}) / V(r_{v,t}), \\ COV &= 2Cov\left(\sum_{i=1}^p \phi_{i,1}^* \cdot r_{S,t-i} + \sum_{i=1}^p \phi_{i,2}^* \cdot r_{v,t-i}, \beta^* r_{S,t}\right) / V(r_{v,t}), \\ VE &= V(\varepsilon_t^*) / V(r_{v,t}). \end{aligned}$$

VL , VCR , COV and VE measure the percentage of the overall variance of $r_{v,t}$ explained by lagged DAX and volatility returns (VL), contemporaneous DAX returns (VCR), covariance between lagged DAX and volatility returns and contemporaneous DAX returns (COV) and variance of the residuals (VE).

In the first three columns of Table 5, we report the sampling frequency, the number of lags employed¹² and the number of valid observations. The p -values 1 and 2 refer to a test of the

¹² The number of lags is taken to be alternatively $p = 1$ or the optimal choice indicated by the Akaike and Schwartz criterion. For hourly and daily data, the latter choice is equal to $p = 1$.

hypothesis that the MSE of a forecast of $r_{v,t}$ based on the unrestricted model is the same as the MSE based on restricted models 1 and 2, respectively. In the case of restricted model 1, this hypothesis is always rejected at the 99% confidence level. Thus, adding $r_{S,t}$ to the set of regressors improves the explanatory power of the model. This finding confirms that part of the relationship occurs contemporaneously. In the second comparison we test whether the model with lagged and contemporaneous returns (unrestricted model) has additional explanatory power above restricted model 2 which only uses contemporaneous index returns. Again, with one exception, all p -values are below 1%. Thus, even after controlling for contemporaneous returns, a significant part of volatility changes can be traced back to leading index returns. For high-frequency data, a substantial part of the variance of $r_{v,t}$ can be attributed to leading returns (VL). At lower frequencies, the lead-lag-relationship is negligible, and the variation of $r_{v,t}$ is primarily attributed to contemporaneous index returns (VCR).

4.3 Impulse-response functions

As a natural extension of the Granger causality analysis, we use impulse-response functions (IRFs) to illustrate the dynamic relations between DAX and implied volatility returns. An IRF describes the impact of a one-time impulse in one variable on future values of the other variable. It allows us to assess how important the impact is and how long it lasts. More precisely, let subscripts i and j refer to DAX and volatility returns. We denote by s a forecast period starting from date t (forecast horizon $t + s$) and assume that the state of the system as of date t is known. Then, IRF is a function of s whose values correspond to the revision in the forecast of $r_{i,t+s}$ induced by the information that the value of $r_{j,t+s}$ is higher than expected ($\varepsilon_{j,t} > 0$) (see Hamilton (1994), p. 318-323).

Theoretically, IRFs can be obtained from the coefficients of the vector MA(∞) representation of the original VAR:

$$\mathbf{R}_t = \mathbf{c} + \boldsymbol{\varepsilon}_t + \boldsymbol{\Psi}_1 \cdot \boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\Psi}_2 \cdot \boldsymbol{\varepsilon}_{t-2} + \dots \quad (8)$$

The row i , column j ($i, j \in \{1, 2\}$) element of $\boldsymbol{\Psi}_s$ is the response of the i th variable at time $t + s$ to a one-time shock on the j th variable at time t . We obtain estimates of $\boldsymbol{\Psi}_s$ by simulation of the original VAR (3)¹³ (see Hamilton (1994), p. 319, for details).

If the innovations $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are correlated, knowledge of $\varepsilon_{j,t}$ has implications for the distribution of $\varepsilon_{i,t}$. These implications are relevant for revising the expectation of $r_{i,t+s}$. Hence, setting $\varepsilon_{i,t}$ to zero in the simulations would lead to misleading IRFs. Clearly, the innovations $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ in the returns regression (4) and the volatility changes regression (5) are indeed correlated (see Section 3). Therefore, we orthogonalize the impulses which ensures that direct and indirect effects of an impulse are considered.¹⁴

¹³ As before, we estimate the autoregressive coefficients of the VAR system 3 by OLS with a maximal lag p selected according to the Akaike and Schwartz information criteria. In the simulation, we set $\mathbf{R}_{t-1} = \mathbf{R}_{t-2} = \dots = \mathbf{R}_{t-p} = \mathbf{0}$, $\varepsilon_{j,t} = 1$ and $\varepsilon_{i,t} = 0$ and simulate the system (3) for times $t, t + 1, t + 2, \dots, t + q$.

¹⁴ The orthogonalization is based on a triangular factorization of the estimate of the variance-covariance matrix Ω of $\boldsymbol{\varepsilon}_t$. See Hamilton (1994), p. 320, for details.

10 Days < Time-to-Maturity \leq 1 Month								
p	#data	p -value 1	p -value 2	VL	VCR	COV	VE	
5m.	1	39137	<0.0001	<0.0001	2.17%	4.22%	0.38%	93.22%
	5	11302	<0.0001	<0.0001	6.55%	4.53%	0.67%	88.25%
15m.	1	11138	<0.0001	<0.0001	1.20%	14.36%	0.60%	83.84%
	3	4671	<0.0001	<0.0001	9.55%	7.75%	0.47%	82.23%
1h.	1	598	<0.0001	0.0009	0.19%	41.90%	-0.02%	57.92%
1d.	1	1591	<0.0001	0.5923	0.24%	40.09%	-0.03%	59.70%
1 Month < Time-to-Maturity \leq 2 Months								
p	#data	p -value 1	p -value 2	VL	VCR	COV	VE	
5m.	1	12070	<0.0001	<0.0001	1.74%	2.91%	0.09%	95.27%
	4	2706	<0.0001	<0.0001	9.58%	2.81%	-0.18%	87.79%
15m.	1	3272	<0.0001	<0.0001	1.91%	10.06%	0.10%	87.93%
	2	1668	<0.0001	<0.0001	4.84%	7.40%	0.58%	87.17%
1h.	1	746	<0.0001	0.0013	1.20%	14.36%	0.60%	83.84%
1d.	1	2774	<0.0001	0.0185	0.19%	41.90%	-0.02%	57.92%

Table 5: Analysis of variance for the total sample period from 1995 to 2005. The first two columns display the sampling frequency and the number of lags employed in the regressions. The third and fourth columns report the p -value of testing whether the MSE of the unrestricted model is the same as the MSE of the restricted model 1 or 2 (p -value 1 and p -value 2). The last four columns report the portion of volatility returns' variance explained by the various subsets of regressors.

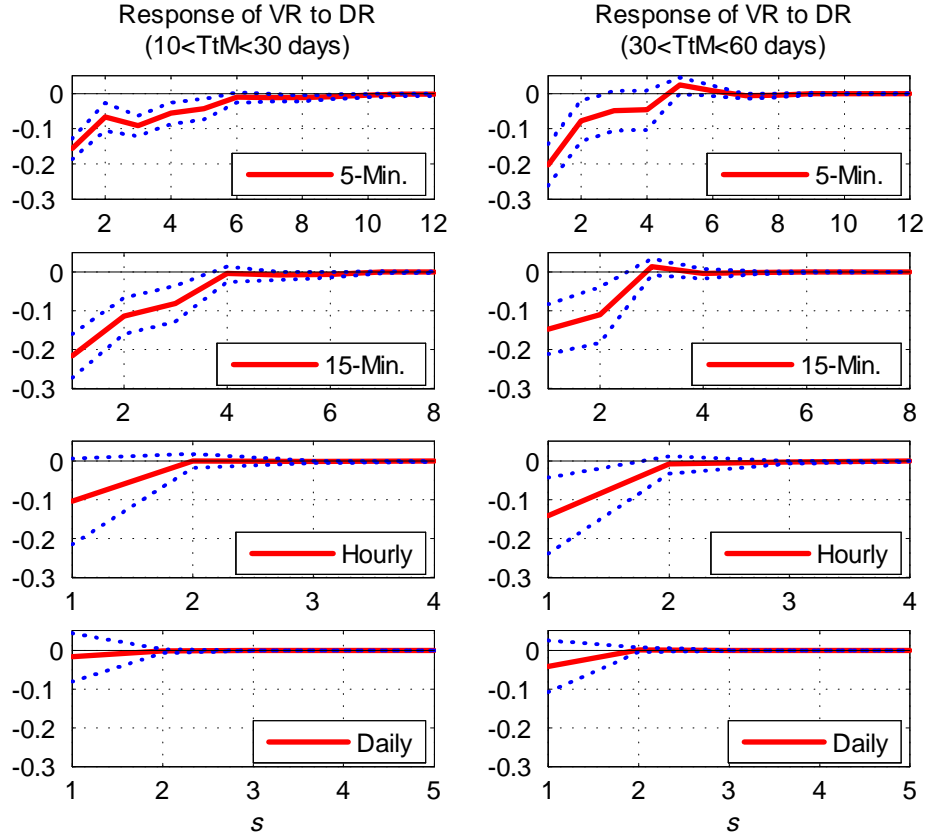


Figure 2: Orthogonalized impulse responses of volatility returns (VR) to DAX returns (DR). The panels on the left (respectively on the right) display the IRFs for the volatility computed from options with the a time-to-maturity ranging from 10 to 30 days (30 to 60 days).

Figures 2 and 3 illustrate the orthogonalized IRFs for DAX and volatility return innovations. The magnitude of the shock is fixed at one standard deviation of the uncorrelated (orthogonalized) innovation. We add two-standard-error bands from Monte Carlo simulations with 100'000 paths. The horizon s varies from 4 to 12 intervals depending on the sampling frequency. This corresponds to a range of one hour (5-minute data) to 5 days (daily data). The units on the vertical axis are in DAX or volatility return standard deviations.

The IRFs for responses of volatility return to an impulse of DAX return are typically negative (see Figure 2). For 5- and 15-minute data, the responses directly after a shock have a magnitude of about -0.1 to -0.2 standard deviations. The IRFs then remain significantly negative for about 15 to 45 minutes. As expected, the responses are less important for lower frequencies. Figure 3 shows that the impact of a volatility shock on DAX returns is very limited. This observation is compatible with our findings of Section 4.

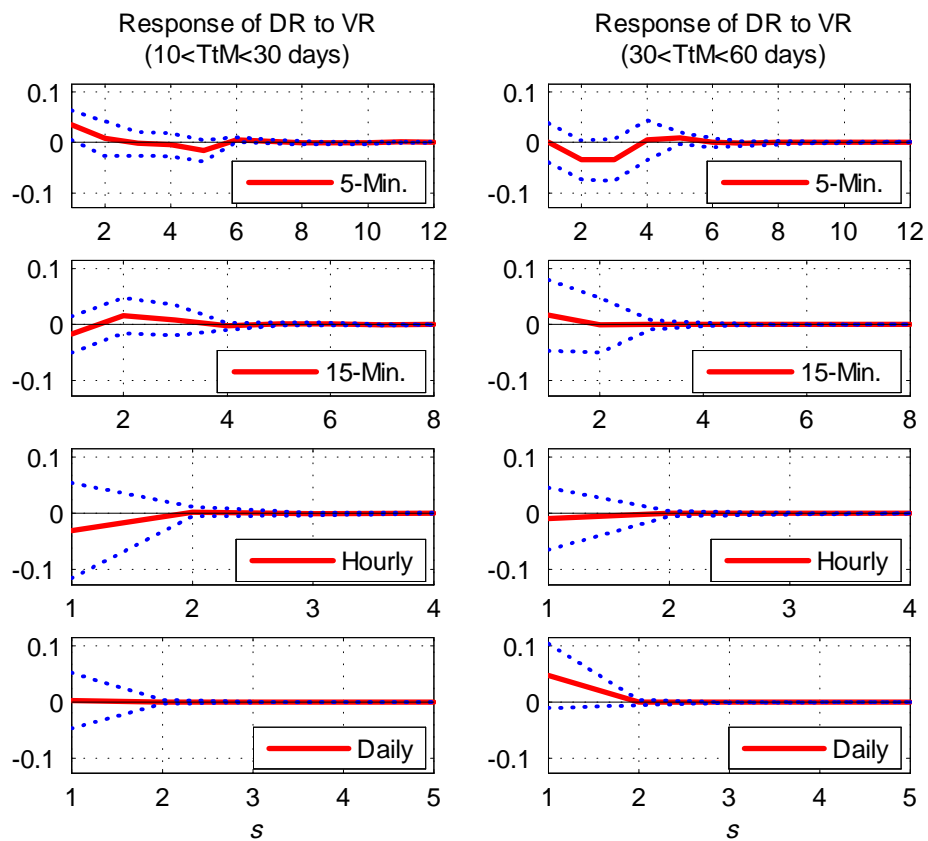


Figure 3: Orthogonalized impulse responses of DAX returns (DR) to volatility returns (VR). The panels on the left (respectively on the right) display the IRFs for the volatility computed from options with the a time-to-maturity ranging from 10 to 30 days (30 to 60 days).

5 Conclusion

It is well known that index returns are inversely related to volatility changes. The relationship is so strong that it constitutes an important stylized fact in finance. Nevertheless, the origin and the causes of the effect are not yet well understood, which is particularly true for financial markets in Europe. In this paper, we analyze the return-volatility relationship at the German market. We calculate return series for 5-minute intervals from tick-by-tick DAX option and futures data over the time period from 1995 to 2005. We also consider lower return frequencies for the purpose of comparison. Our volatility measure is the implied volatility of at-the-money (ATM) options. This allows us to more accurately detect changes in volatility than previous studies which use measures of realized volatility. In addition, as ATM implied volatilities can be determined independently of the underlying asset return, index and volatility returns can be modelled jointly in a VAR model. This provides a flexible framework for running Granger causality tests and computing impulse-response functions.

We find that the contemporaneous inverse relationship in high-frequency data is linear without any systematic differences between rising and falling markets. A lead-lag relationship exists only in high-frequency data. The relationship is return-driven in the sense that index returns Granger cause volatility changes. This causal relationship is statistically and economically significant and can be clearly separated from the contemporaneous correlation. A volatility feedback effect does not show up. Either it does not exist, or the market promptly incorporates all direct and indirect impulses into market prices so that the feedback effect fully evolves within the 5-minute intervals.

Our paper does not account for jumps either in the index level or the (implied) volatility process. The relationship around such discontinuities could offer further insight into the nature of the effect. The behavior of the relationship through time could also be of interest, because it is closely related to the pricing of volatility and the dynamics of the volatility risk premium. But the most important topic for further research still seems to be the question why this strong effect exists and which economic fundamentals are effectively driving it.

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